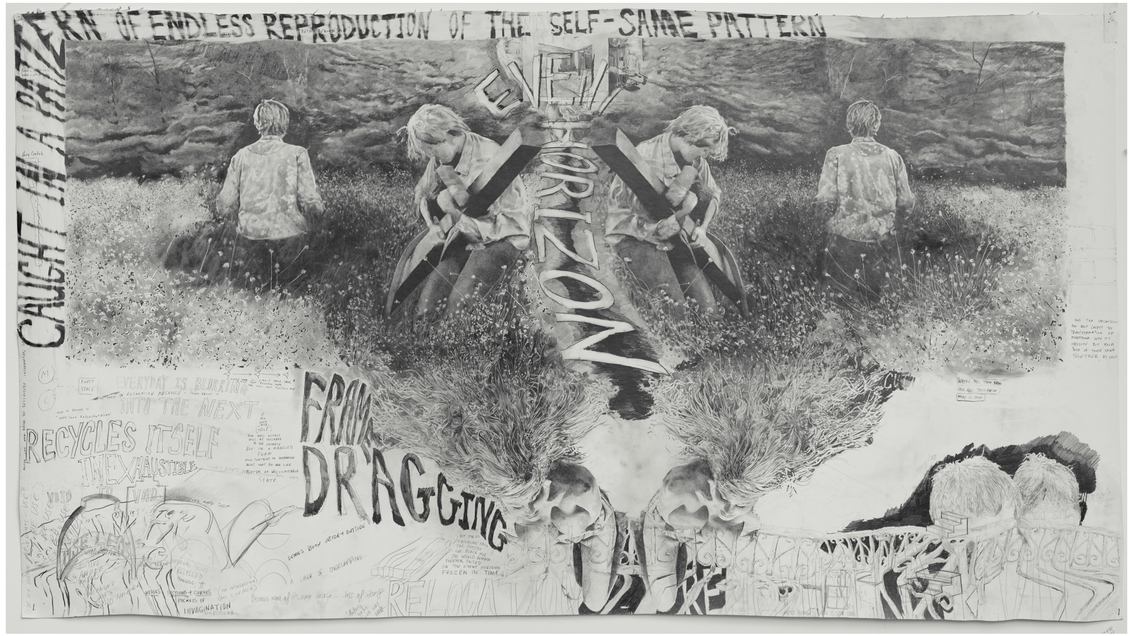
Gravitational Waves from Viscous Stars



Event Horizon, 2017 (Kaari Upson)

Jaime Redondo-Yuste

ICC, Universitat de Barcelona

(Loosely) based on 2411.16841 (CQG), 2411.16861 (PRL), 2506.13850 (PRD)

THE CENTER OF GRAVITY

KØBENHAVNS UNIVERSITET

Danmarks Grundforskningsfond
Danish National Research Foundation

Outline

Overture

Part I: Dissipative Hydro

Part II: Stellar Perturbations

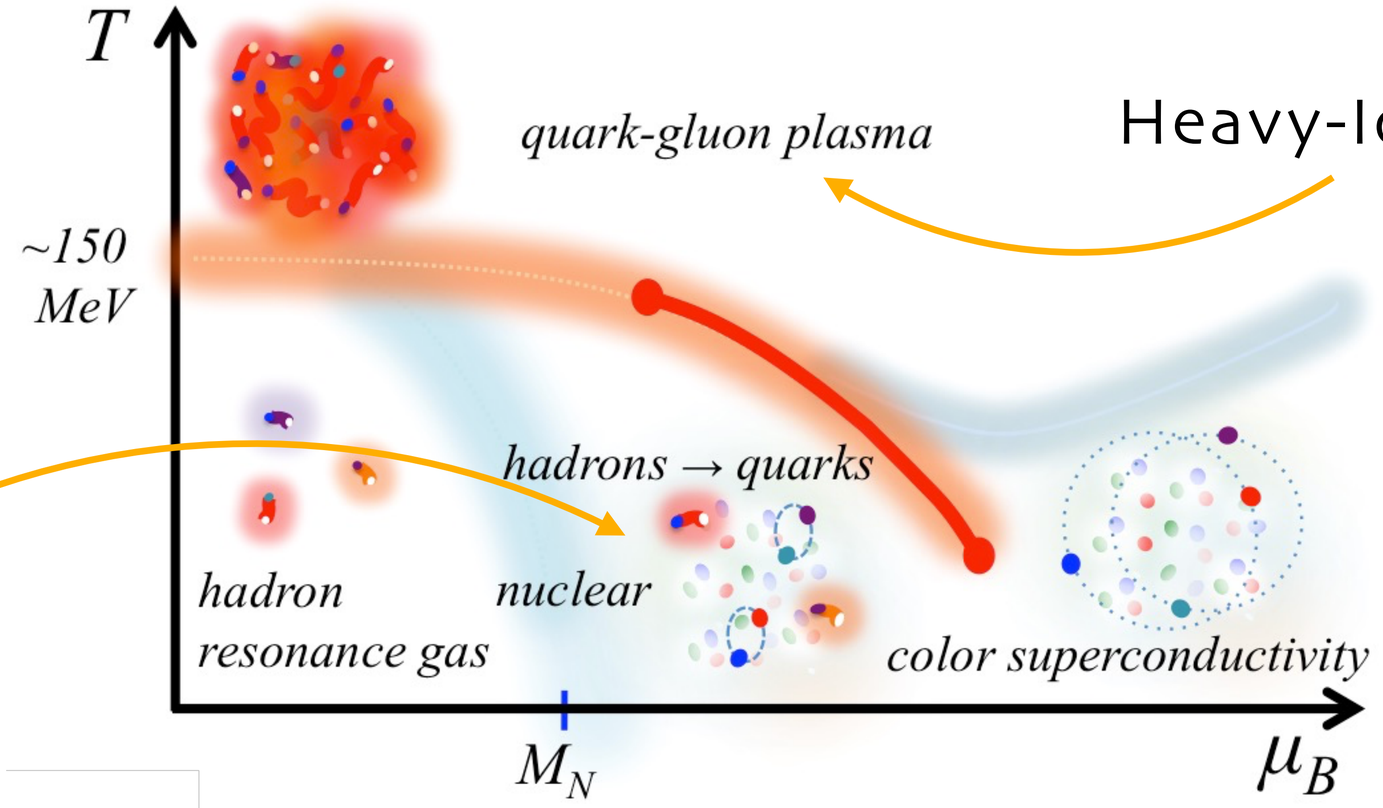
Coda

Overture

Boys bathing at Skågen, P.S.Krøyer

The Standard Model is not the end

Do we understand matter beyond nuclear density?



Heavy-Ion Collisions

[This is expensive]

Neutron Stars

[This is ~~cheaper~~]

Baym+ (2017)

Neutron Stars are layered

Neutron Stars probe matter at different scales

Crust

Outer:

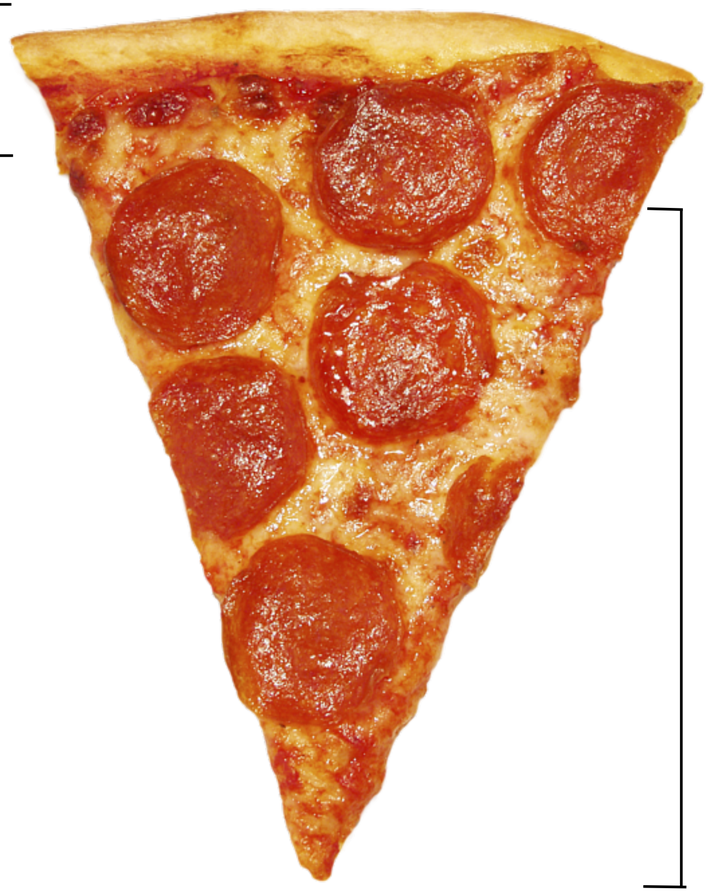
$n_b < 10^{-3} n_{\rm sat}$

$R \lesssim 0.1{\rm km}$

Inner:

$n_b < 0.5 n_{\rm sat}$

$R \lesssim 0.5{\rm km}$



Core

Outer:

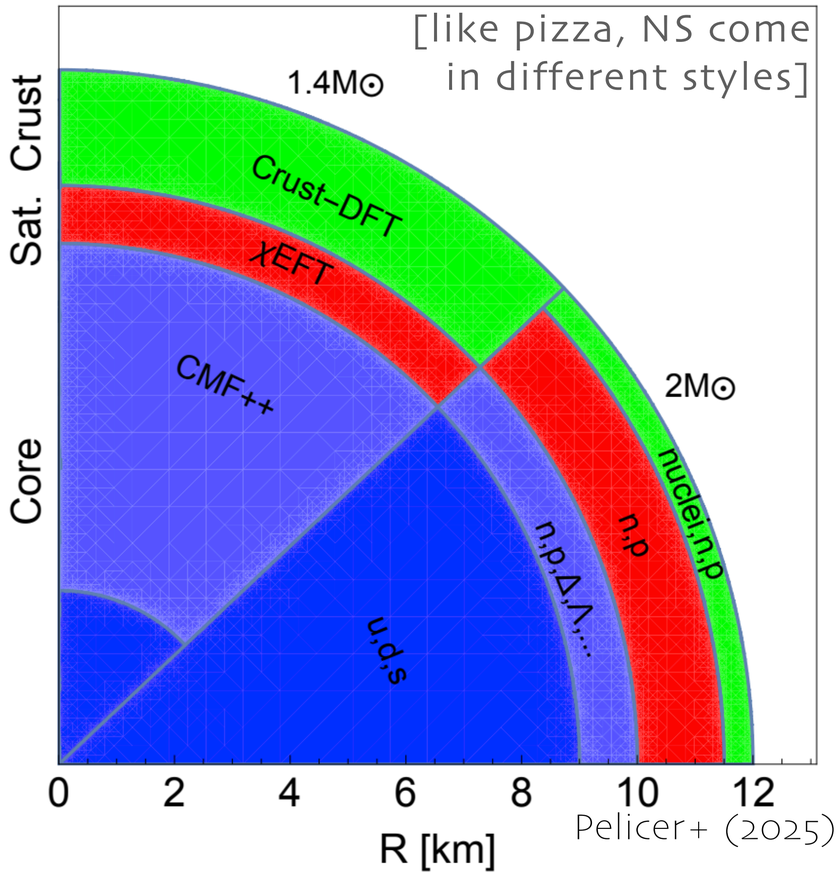
$n_b < 2 n_{\rm sat}$

$R \lesssim 7{\rm km}$

Inner:

$n_b < 10 n_{\rm sat}$

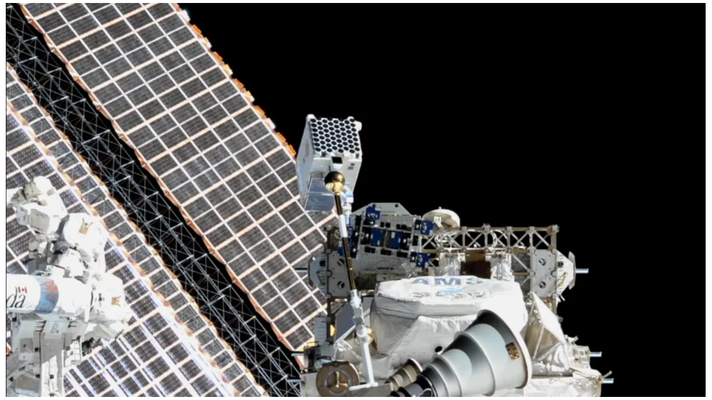
$R \lesssim 5{\rm km}$

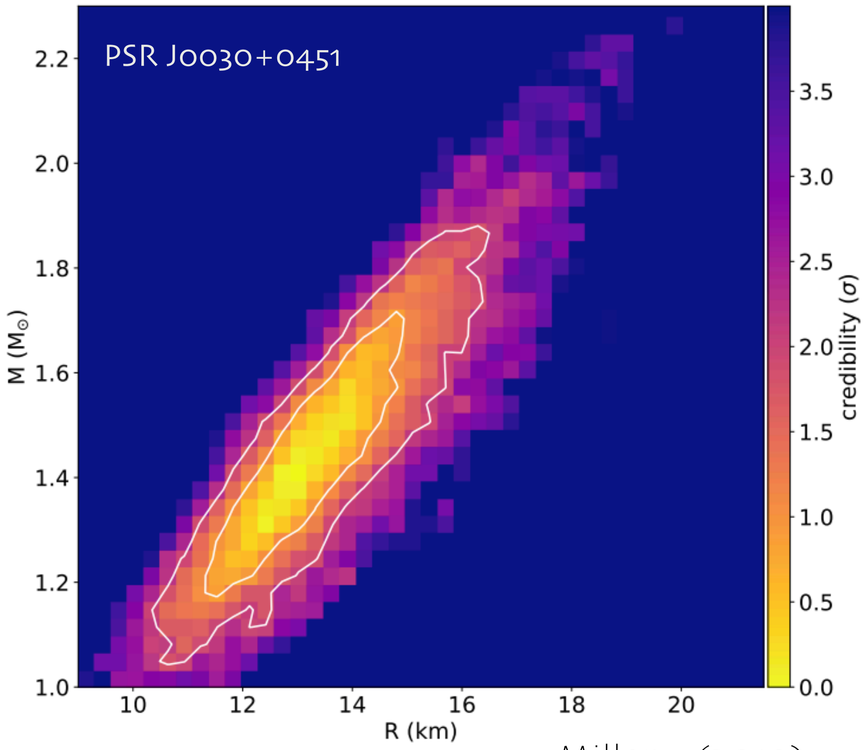


We Observe Neutron Stars with telescopes…

NICER: Mass (5%-level) + Radius (5-20%-level)

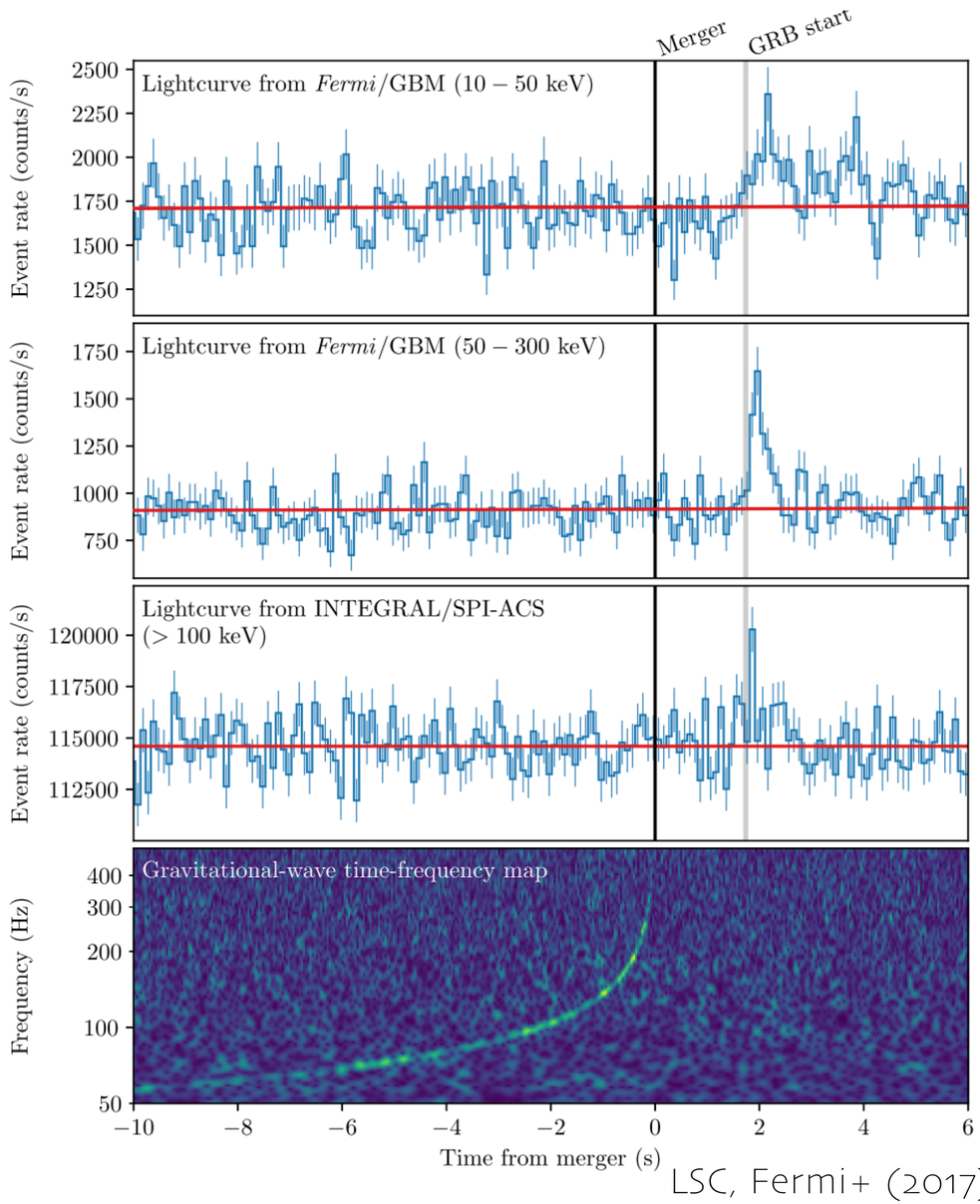
Mass can be better constrained including radio measurements!



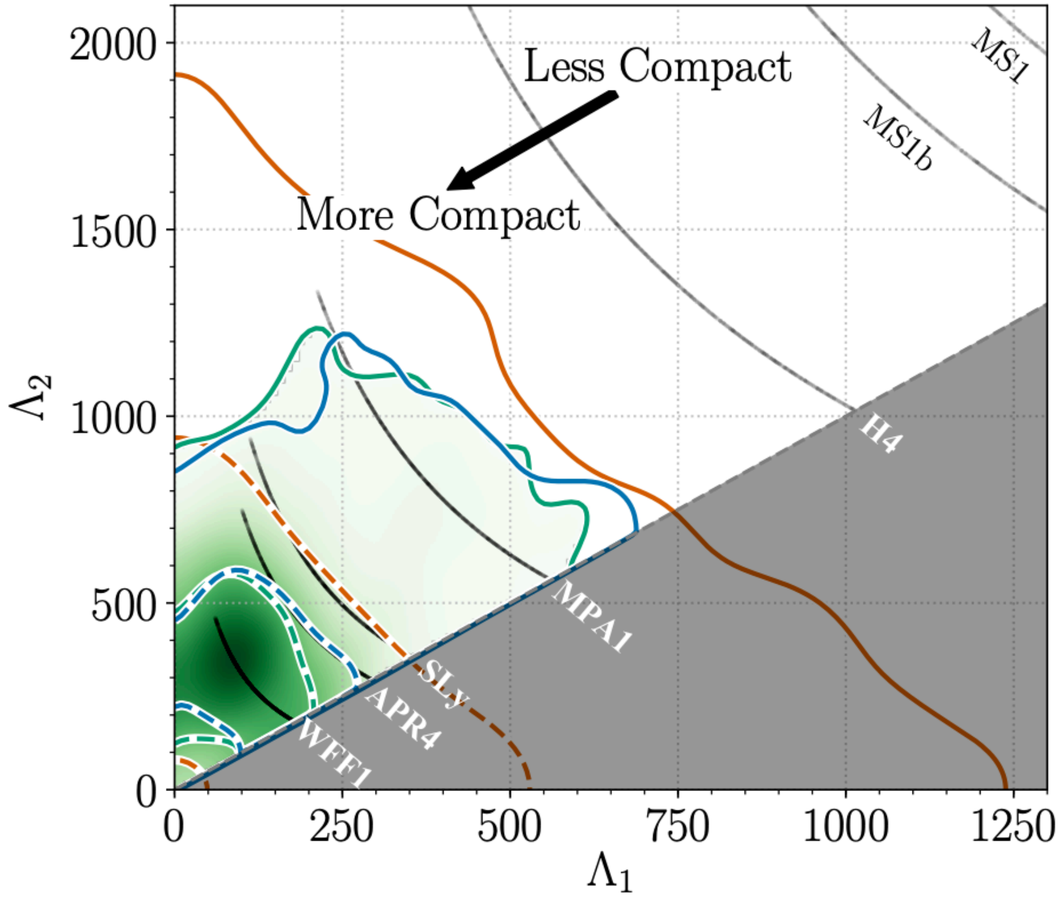


Miller+ (2019)

… and with Gravitational Waves…



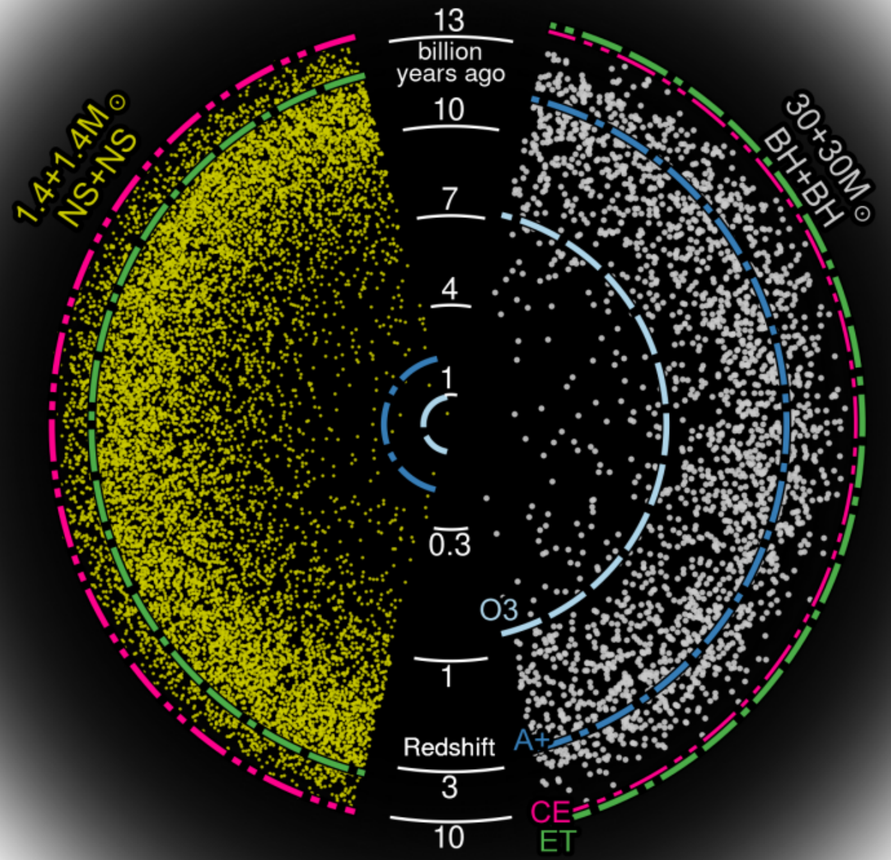
LSC, Fermi+ (2017)



… and we will observe many more!

CE/ET will see
ALL bNS
In
The
Universe

[if they are built]



Cosmology!

Tests of GR

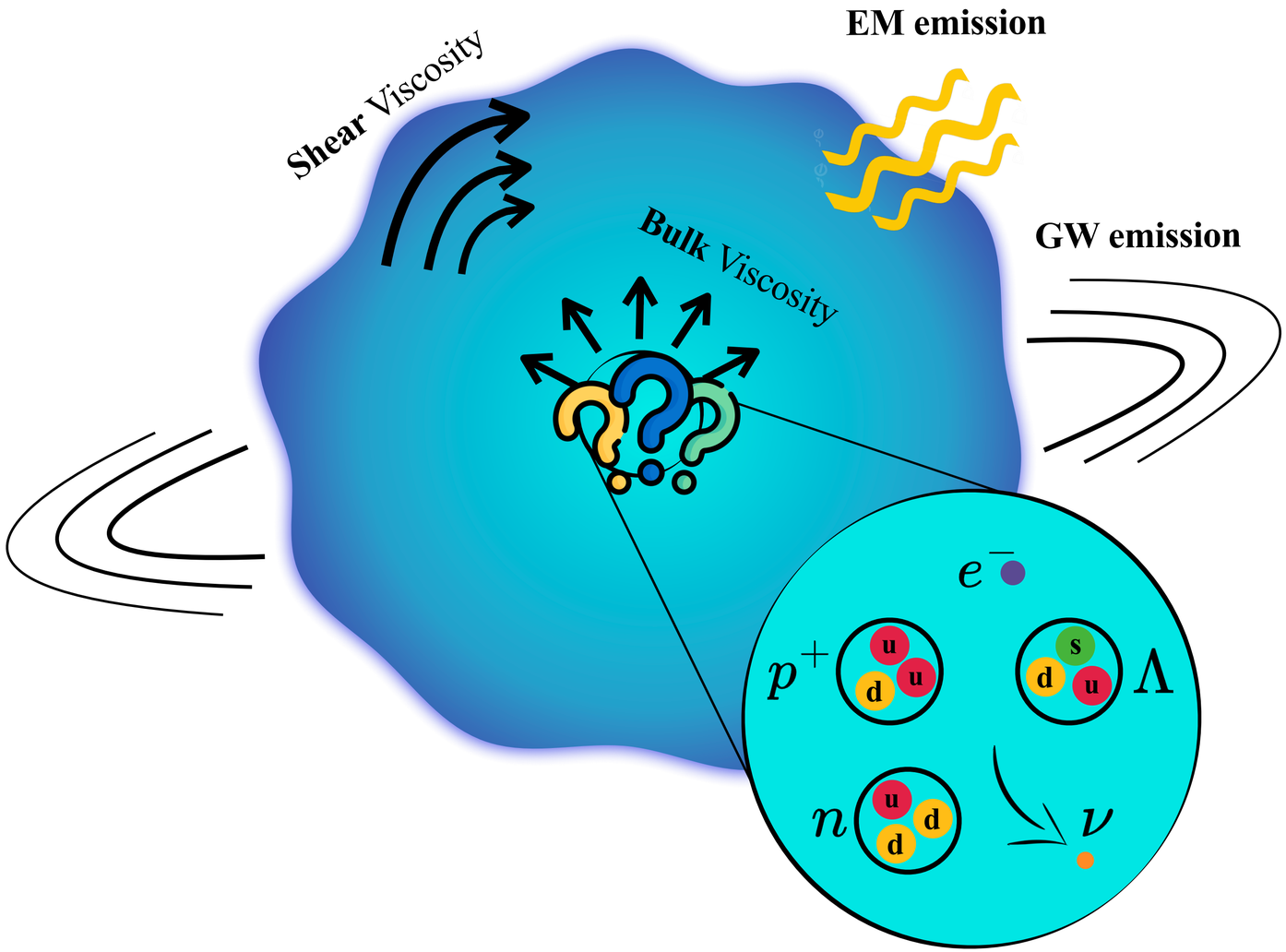
Multimessenger

Heavy Elements

Stellar Populations

…

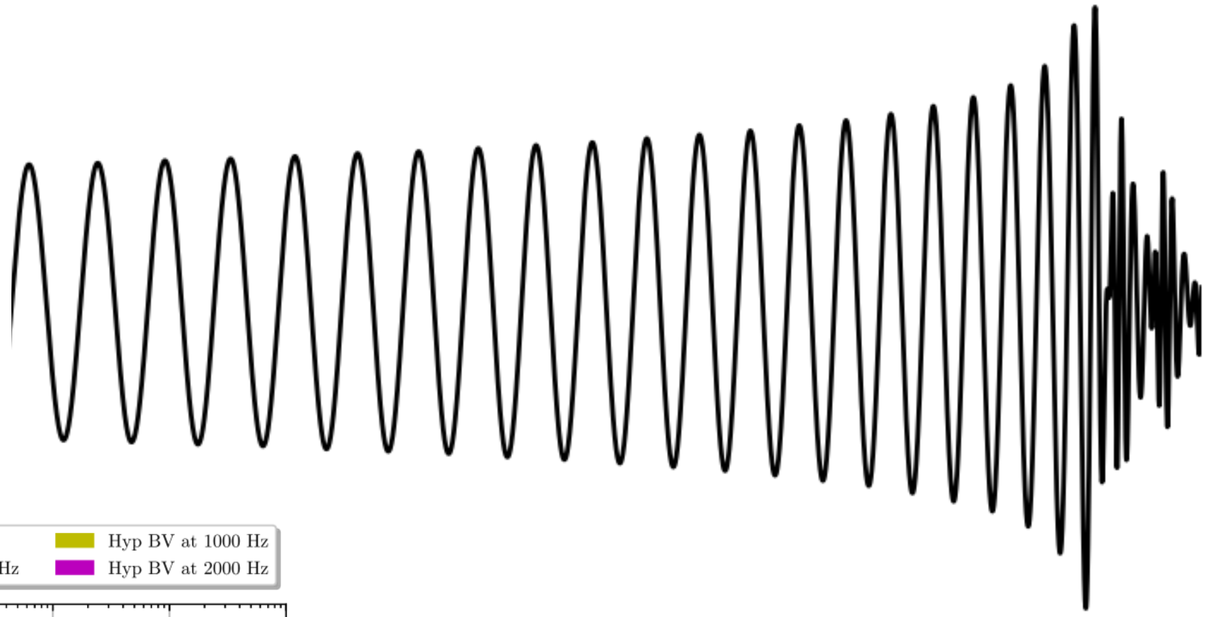
Viscosity: why, oh why?



Viscosity: why, oh why?

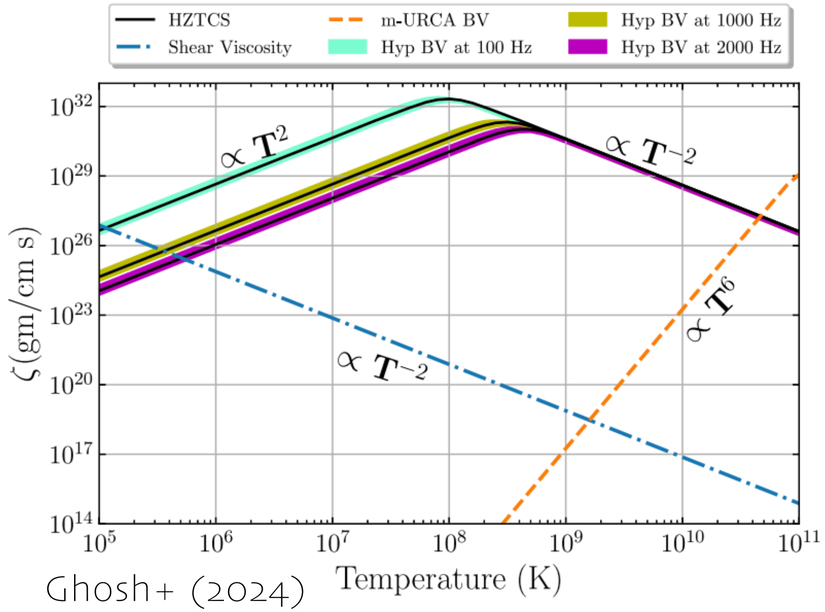
Inspiral

[Can hyperons heat-up enough?]

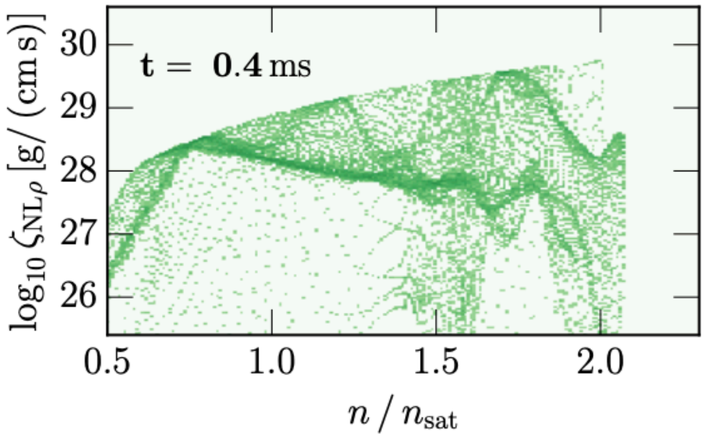


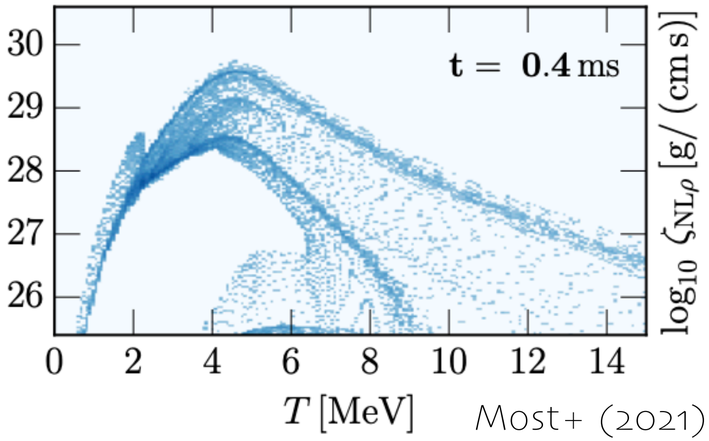
Post-Merger

[Bulk Viscosity (mUrca) matters!]



Ghosh+ (2024)



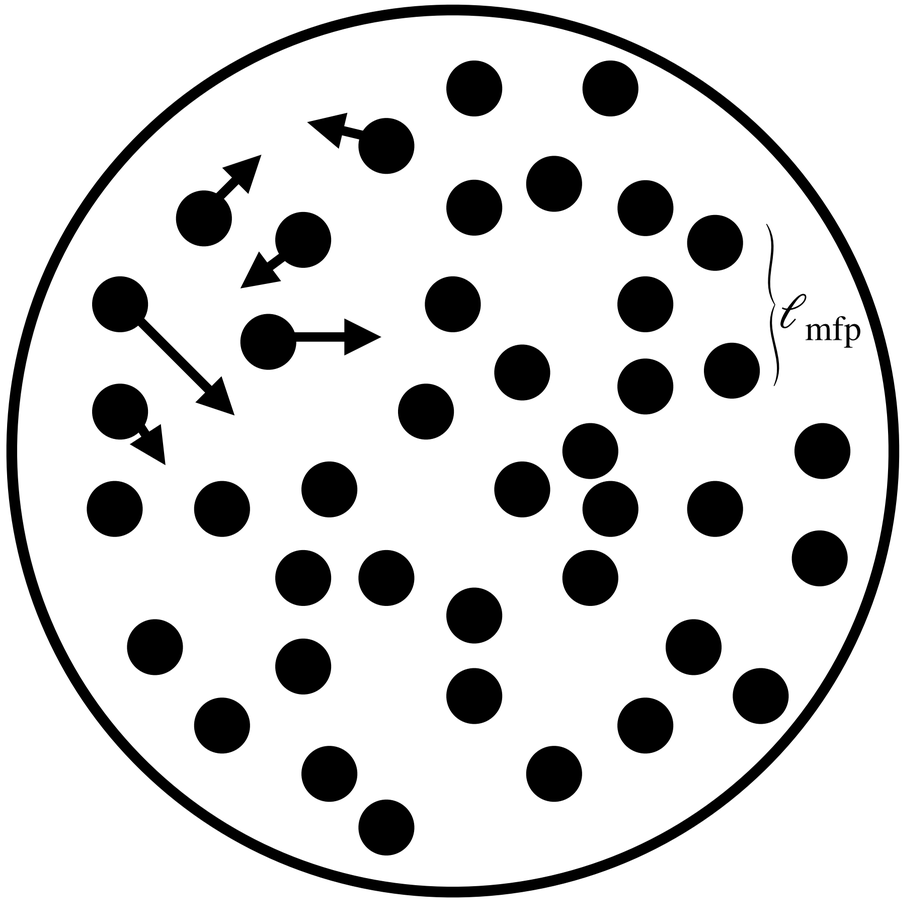


Most+ (2021)

I. Dissipative Hydro

Seascape with Buoy, W. Turner

Hydrodynamics is an EFT



Integrate out dof up to a scale $\mathcal{O}(\ell_{\rm mfp}^0)$

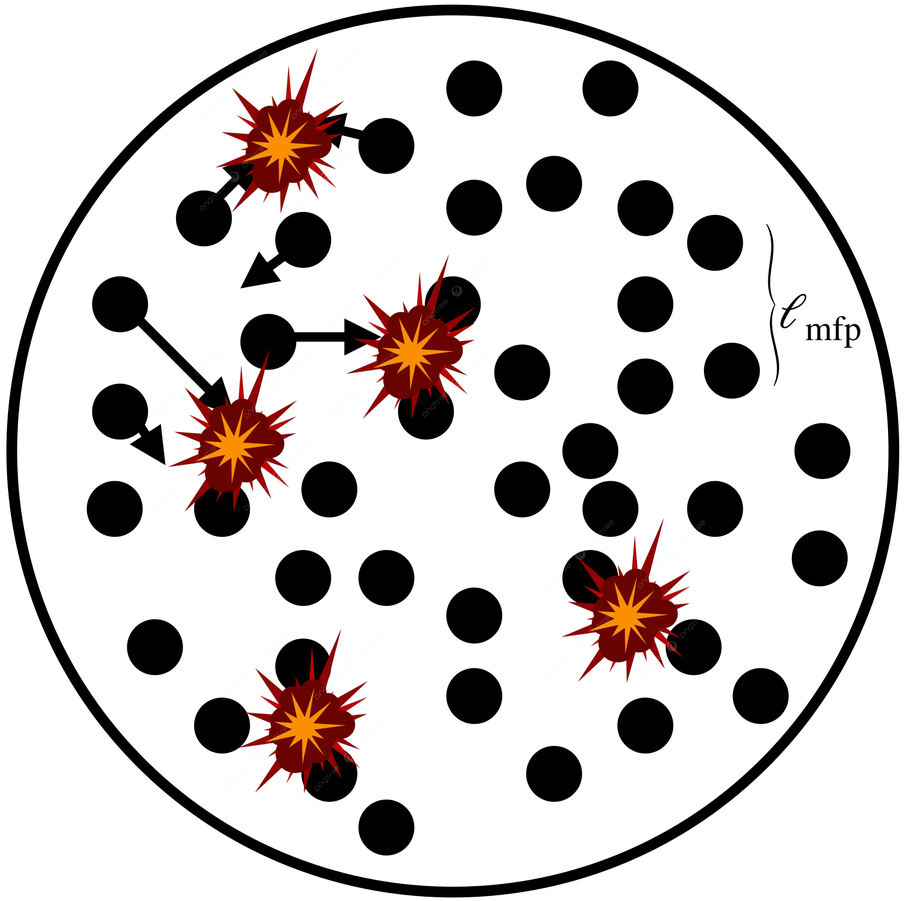
Thermodynamic variables:

$$Z = \mathrm{tr}\exp\Big[\beta(u_a P^a + \mu N)\Big]\,, \quad \beta \equiv T^{-1}$$

Unique rank-2, symmetric tensor:

$$T_{ab} = \varepsilon u_a u_b + p\Delta_{ab}\,, \quad J_a = n u_a$$

First-Order EFT gives Viscosity (I)



Let's keep $\mathcal{O}(\ell_{\rm mfp})$: gradient expansion

$$|\nabla_a X| \ll |X/\ell_{\rm mfp}|$$

Unique rank-2, symmetric tensor:

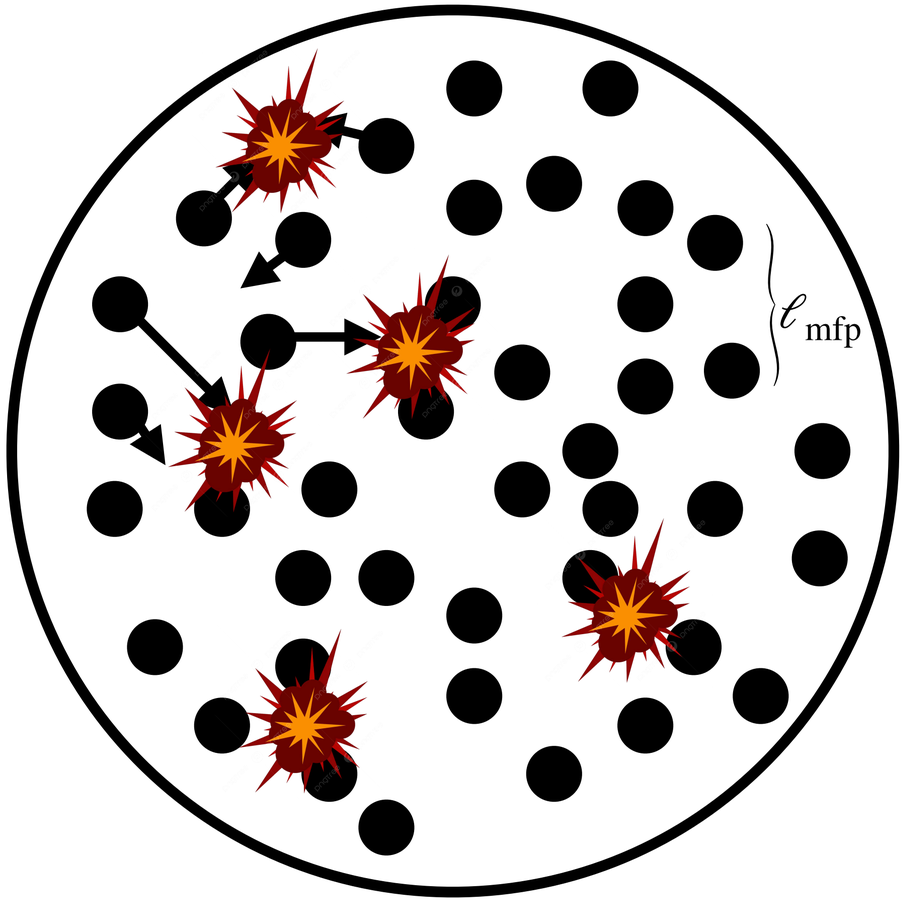
$$T_{ab} = (\varepsilon + \mathscr{E}^{(1)})u_a u_b + (p + \mathscr{P}^{(1)})\Delta_{ab} + 2u_{(a}\mathscr{Q}^{(1)}_{b)} + \mathscr{T}^{(1)}_{ab}$$

$$J_a = (n + \mathscr{N}^{(1)})u_a + \mathscr{J}^{(1)}_a$$

Constitutive Relations:

$$\mathscr{X}^{(1)} \equiv \mathscr{X}^{(1)}(\nabla_a \varepsilon, \nabla_a n, \nabla_a u^b)$$

First-Order EFT gives Viscosity (II)



Look at the entropy current:

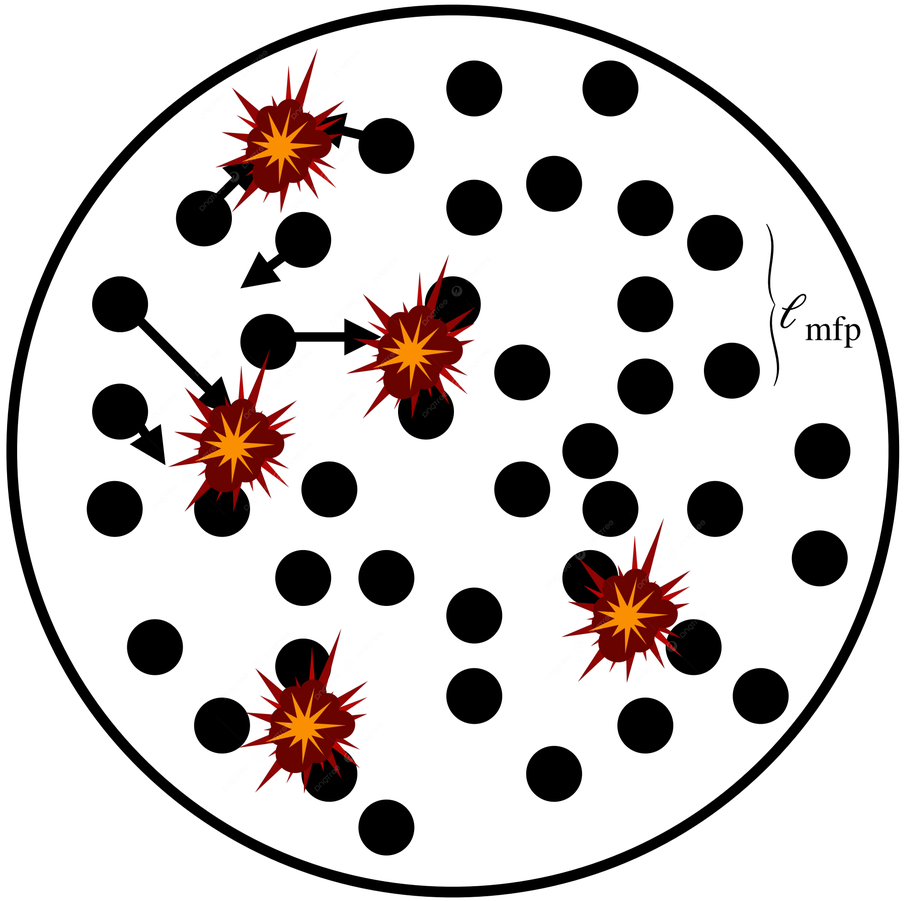
$$TS^a = pu^a - T^{ab}u_b - \mu J^a$$

A straightforward calculation gives

$$\nabla_a S^a = \mathcal{O}(\nabla^2)$$

[This better be non-negative]

First-Order Hydro is Unstable



Hiscock & Lindblom [1985]:

"Any first-order theory satisfying $\nabla_a S^a \geq 0$ *is unstable"*

"These results provide overwhelming motivation (we believe) for abandoning these theories in favor of the second-order (Israel) theories which are free of these difficulties."

BDN [2017-19], Kovtun [2019]:

What if $\nabla_a S^a \geq 0 + \mathcal{O}(\nabla^3)$?

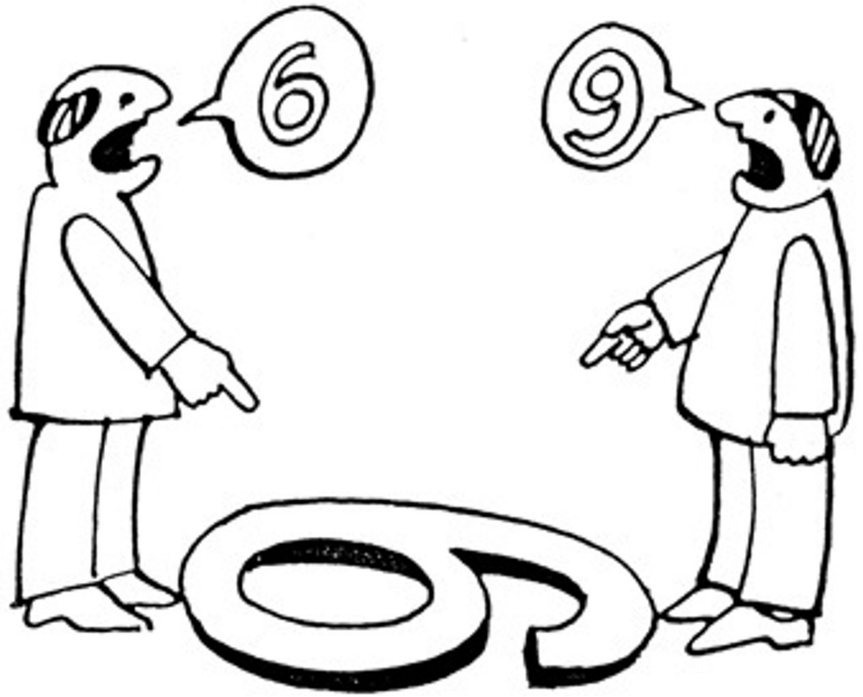
What is a frame, and why does it matter?

Suppose we do a field redefinition
$\varepsilon \to \varepsilon + \alpha u^a \nabla_a \varepsilon$, then:
$\mathscr{E}^{(1)} \to \mathscr{E}^{(1)} + \alpha u^a \nabla_a \varepsilon + \mathcal{O}(\nabla^2)$.

Physically we did nothing

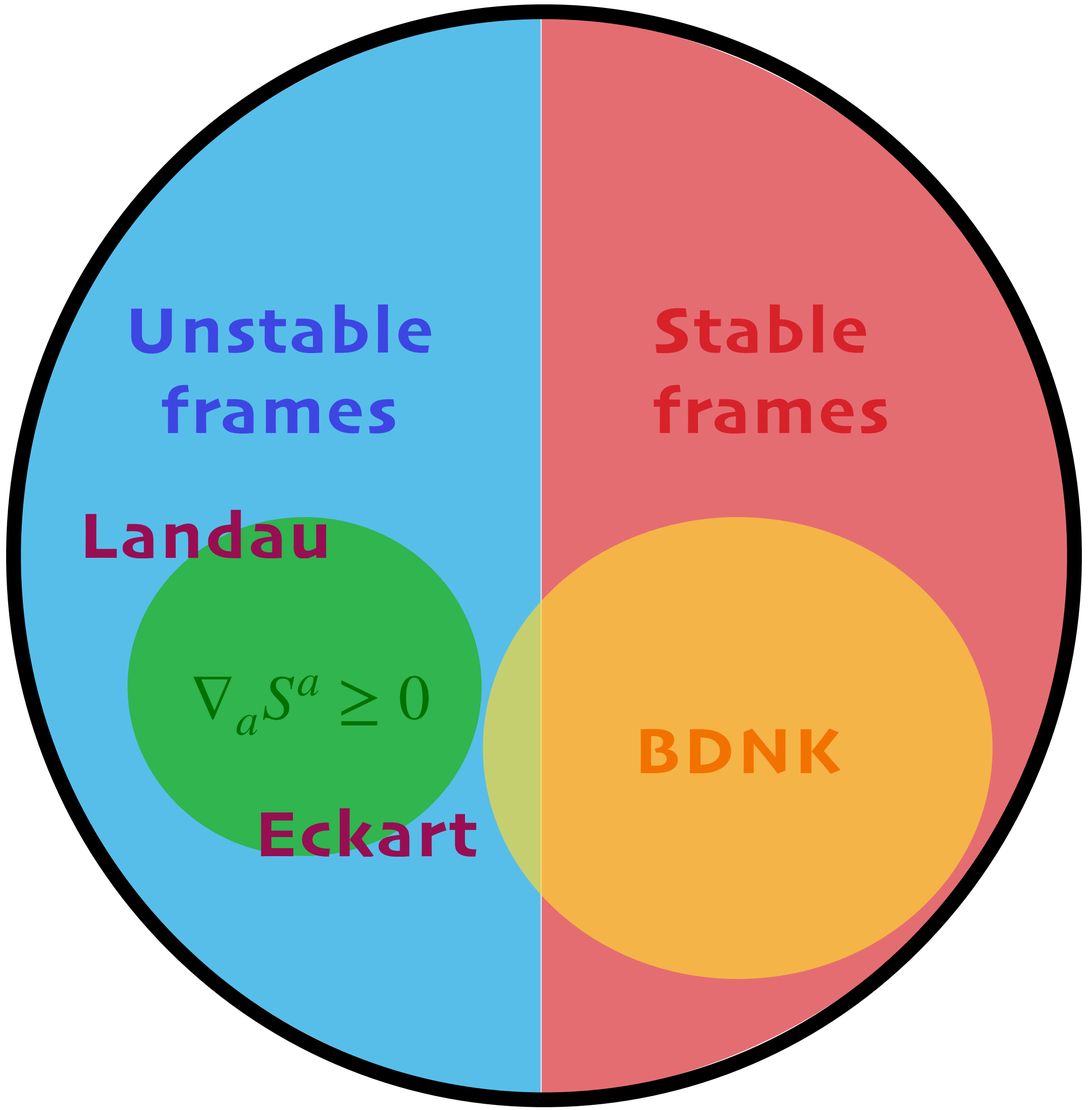
But the equations change!

[Some equations are in Santa's good list,
Some are not]



(For relativists: ADM vs BSSN/CZ4/GHG formulations of GR)

There is a zoo of frames



BDNK Frame has 6 Transport Coefficients

$$\mathscr{E}^{(1)} = \tau_\varepsilon \left[u^a \nabla_a \varepsilon + (\varepsilon + p)\vartheta \right]$$

$$\mathscr{P}^{(1)} = -\zeta\vartheta + \tau_p \left[u^a \nabla_a \varepsilon + (\varepsilon + p)\vartheta \right]$$

$$\mathscr{Q}^{(1)}_a = \tau_Q (\varepsilon + p) u^b \nabla_b u_a + \beta_\varepsilon \Delta_{ab} \nabla^b \varepsilon + \beta_n \Delta_{ab} \nabla^b n$$

[These two are not independent]

$$\mathscr{T}^{(1)}_{ab} = -2\eta\sigma_{ab}$$

$$\mathscr{N}^{(1)} = 0$$

$$\mathscr{J}^{(1)}_a = 0$$

[BDNK ~ generalized Eckart]

Causality implies a number of inequalities between these coefficients

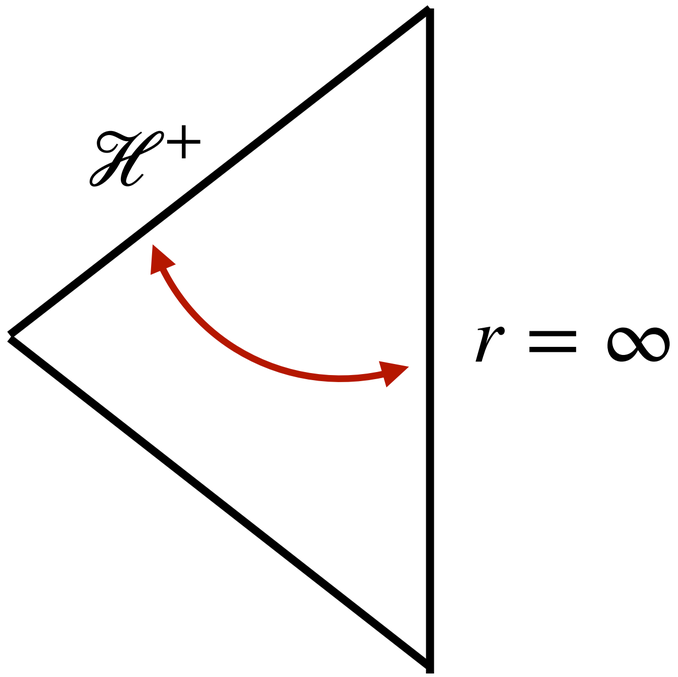
BDNK Frame may "be holographic"

Ciambelli & Lehner (2023)

1st-order in gradient expansion for boosted black brane in aAdS

Require horizon location to be fixed & $\ell_a \parallel u_a$ at the horizon

Then, AdS boundary fluid is (conformal) BDNK!



II. Stellar Perturbations

Untitled (Ocean), V. Celmins

Shaken, not stirred

JRY (2024)

Let us perturb a viscous star!

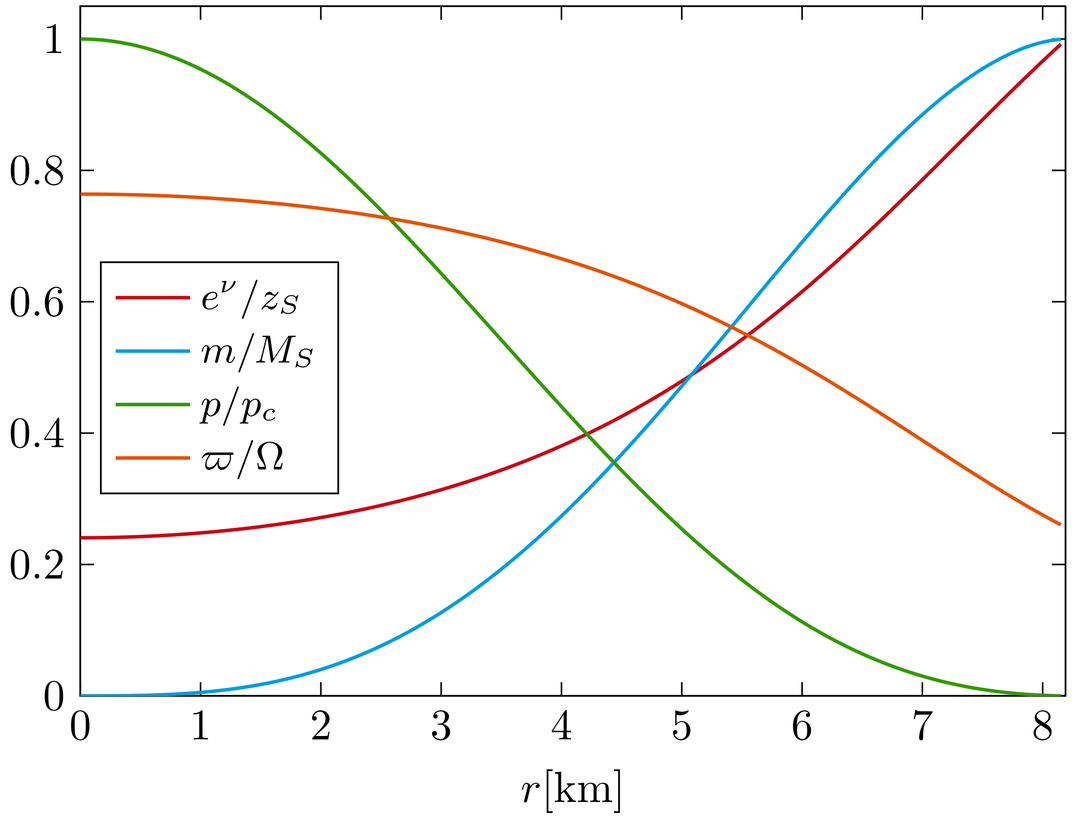
$$g_{ab} = \bar{g}_{ab} + \epsilon h_{ab} \qquad T_{ab} = \bar{T}_{ab} + \epsilon t_{ab}$$

Static + Spherical Symmetry:

$$\bar{g}_{ab}dx^a dx^b = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2,$$

$$u^a = e^{\nu/2}(\partial_t)^a$$

$$e^{\lambda} = \left(1 - \frac{m}{r}\right)^{-1}$$



Different Parities don't talk to each other

JRY (2024)

Spherical Symmetry: SO(3) — Tensor Harmonics

$$h_{ab} = h^{\text{axial}}_{ab} + h^{\text{polar}}_{ab} \qquad t_{ab} = t^{\text{axial}}_{ab} + t^{\text{polar}}_{ab}$$

$$h^{\text{axial}}_{ab} = \begin{pmatrix} 0 & 0 & h_0 X_\theta & h_0 X_\phi \\ 0 & 0 & h_1 X_\theta & h_1 X_\phi \\ \text{sym} & \text{sym} & 0 & 0 \\ \text{sym} & \text{sym} & 0 & 0 \end{pmatrix}$$

$$h^{\text{polar}}_{ab} = \begin{pmatrix} H_0 & H_1 & 0 & 0 \\ \text{sym} & H_2 & 0 & 0 \\ \text{sym} & \text{sym} & r^2 K & 0 \\ \text{sym} & \text{sym} & 0 & r^2 K \sin^2\theta \end{pmatrix} Y_{\ell m}$$

$$t^{\text{axial}}_{ab} \supset \delta u_a = (0, 0, \beta X_\theta, \beta X_\phi)$$

$$t^{\text{polar}}_{ab} \supset \delta\varepsilon, \delta u_a = (\ldots, \gamma Z, \alpha Z_\theta, \alpha Z_\phi)$$

There is a geometric (2x2), gauge-invariant version of this story

Two Wave Equations Rule Axial Perturbations

JRY (2024)

Outside the star, this is just the Regge-Wheeler equation

[w-modes] $$-\partial_{tt}^2\psi + \partial_{r_\star r_\star}^2\psi - V\psi = 16\pi\eta e^{\nu/2}(\partial_t\psi + A\beta)$$

[η-modes] $$-\tau_Q\partial_{tt}^2\beta + \frac{\eta}{\varepsilon + p}\partial_{r_\star r_\star}^2\beta = L_1[\beta] + L_2[\psi]$$

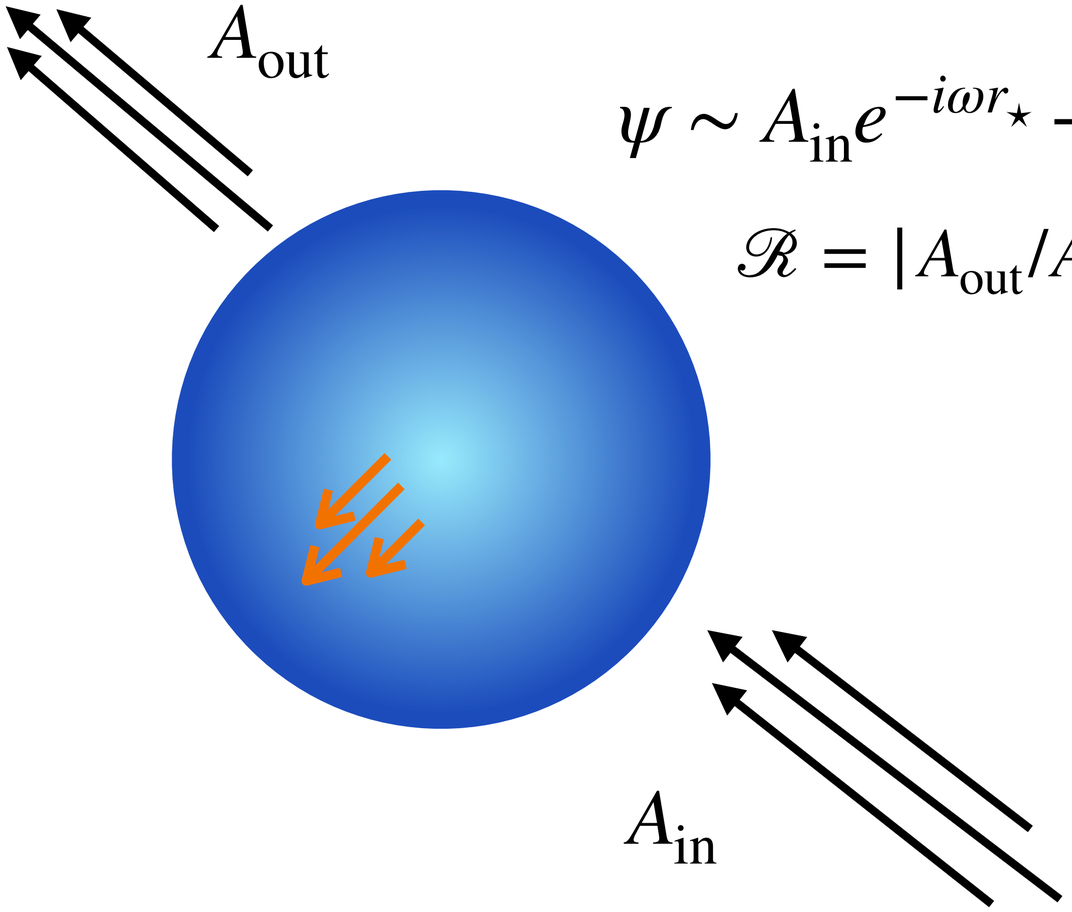
For a perfect fluid, this equation is not dynamical!

Axial + polar eqs are causal iff BDNK Causality Constraints

[Please ask if you want to see the polar equations.
It's not pretty]

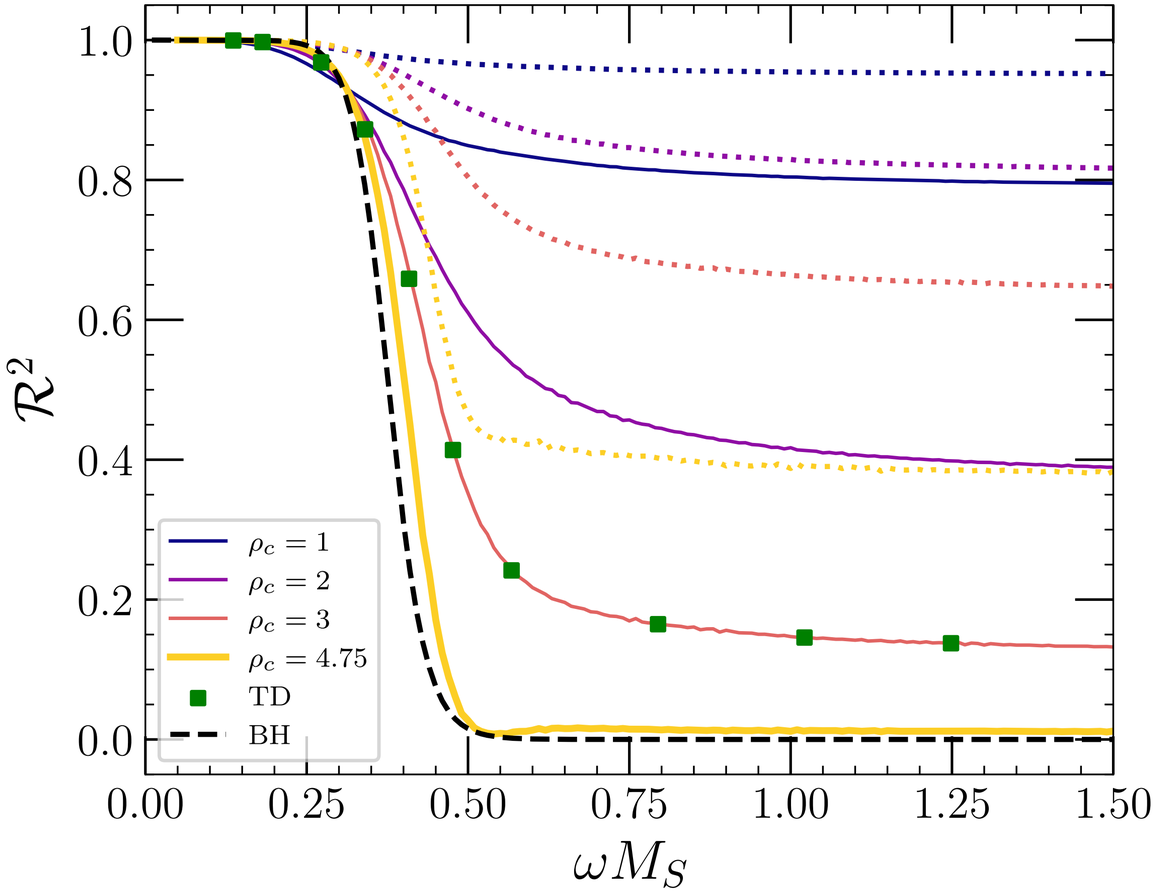
Viscous Stars Absorb GWs (I)

Boyanov, Cardoso, Kokkotas & **JRY (2024)**



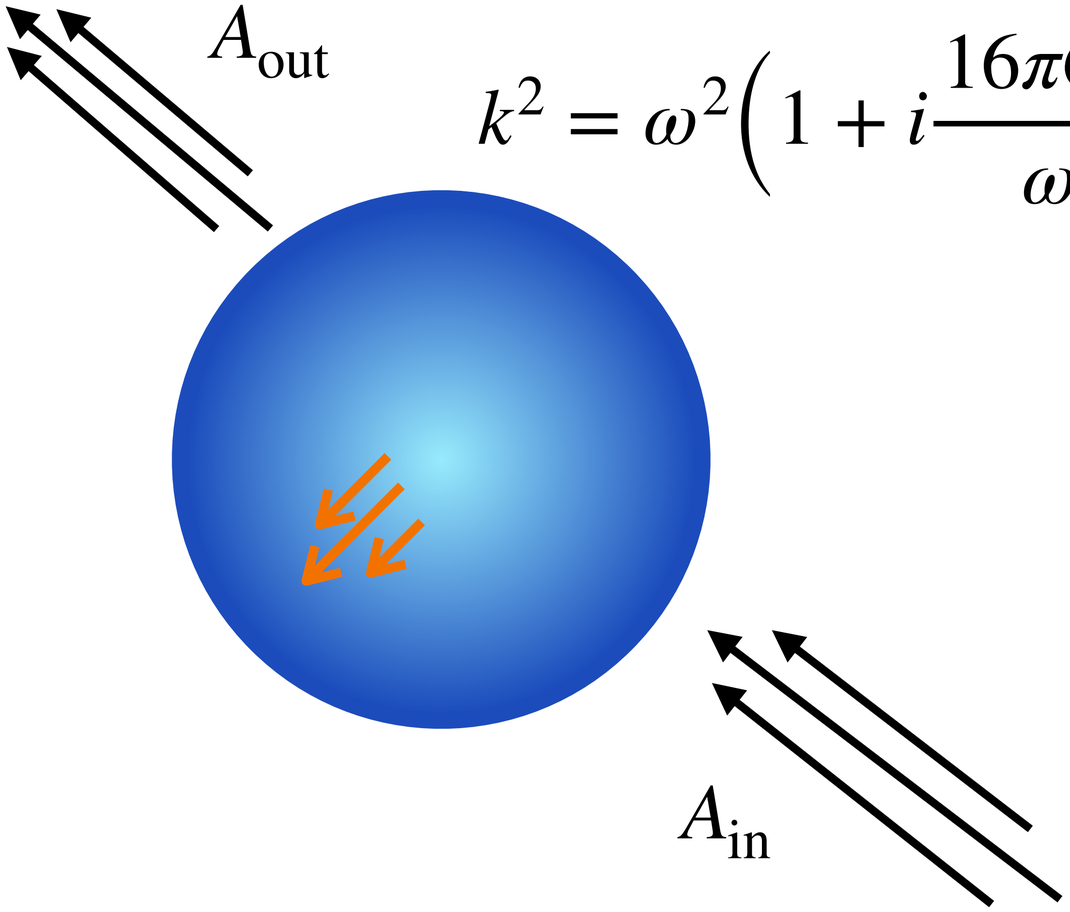
$$\psi \sim A_{\rm in} e^{-i\omega r_\star} + A_{\rm out} e^{i\omega r_\star}$$

$$\mathscr{R} = |A_{\rm out}/A_{\rm in}|^2$$

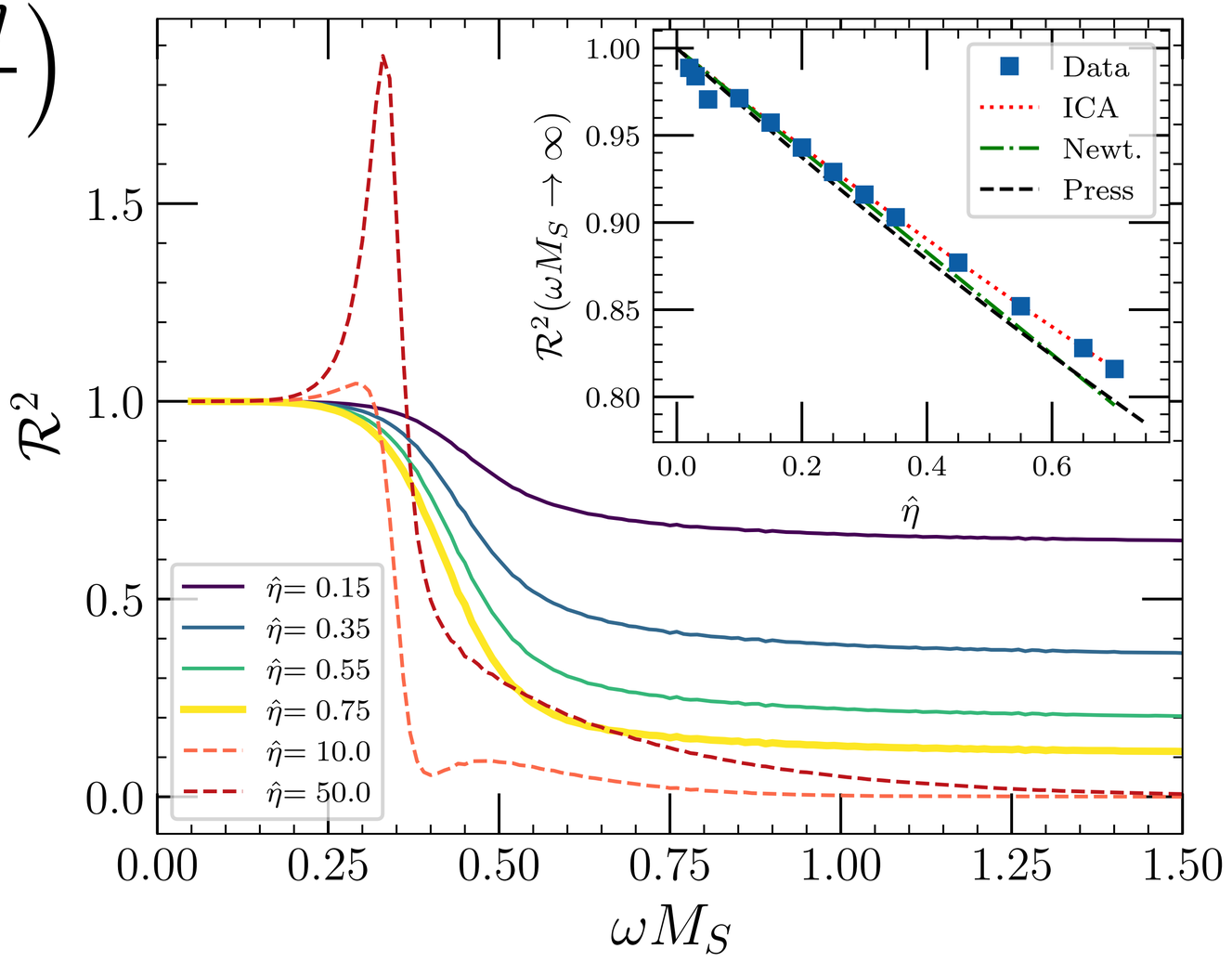


Viscous Stars Absorb GWs (II)

Boyanov, Cardoso, Kokkotas & **JRY (2024)**



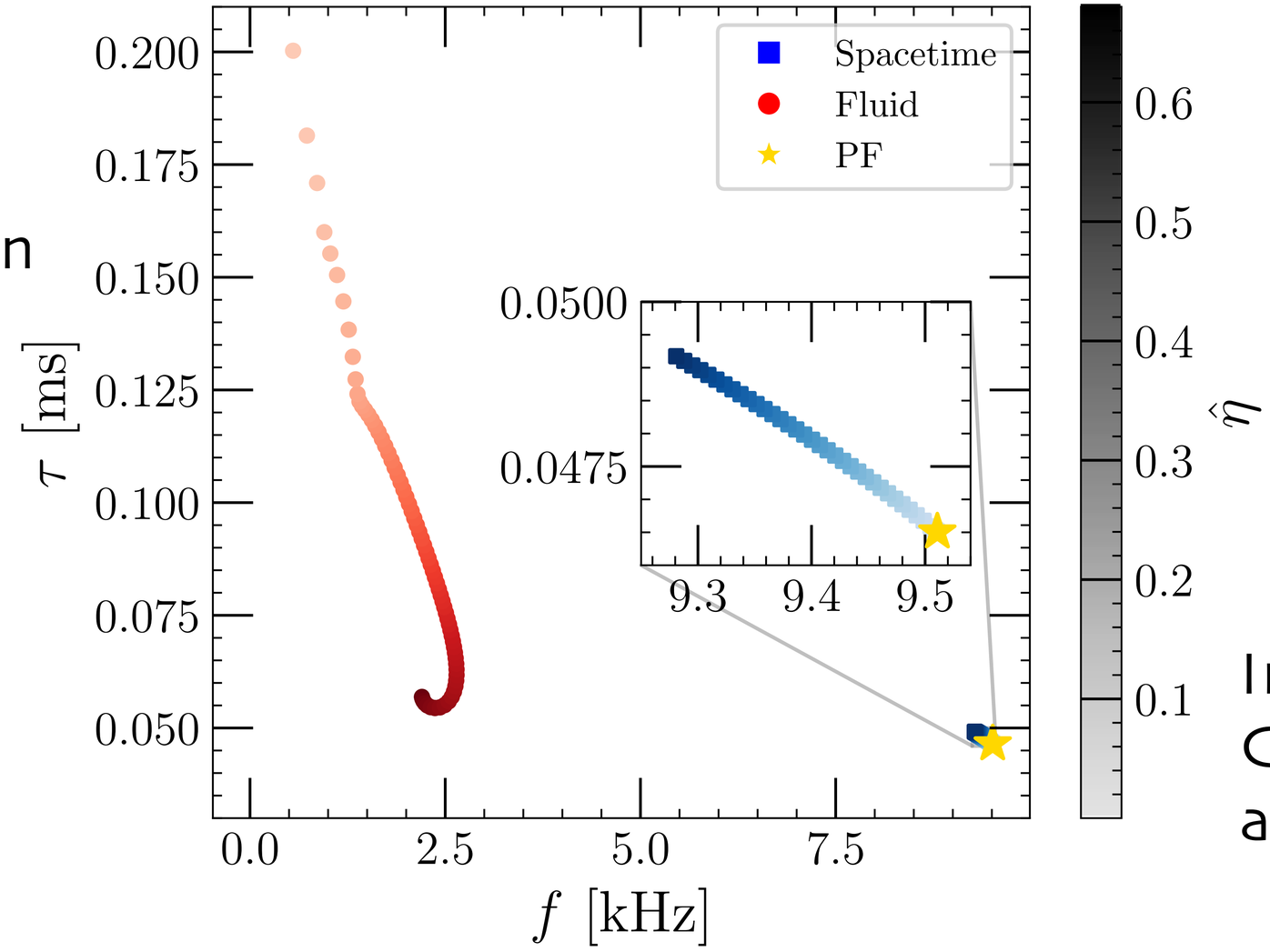
$$k^2 = \omega^2\left(1 + i\frac{16\pi G\eta}{\omega}\right)$$



w-modes are affected by viscosity

Boyanov, Cardoso, Kokkotas & **JRY (2024)**

Cowling-approximation

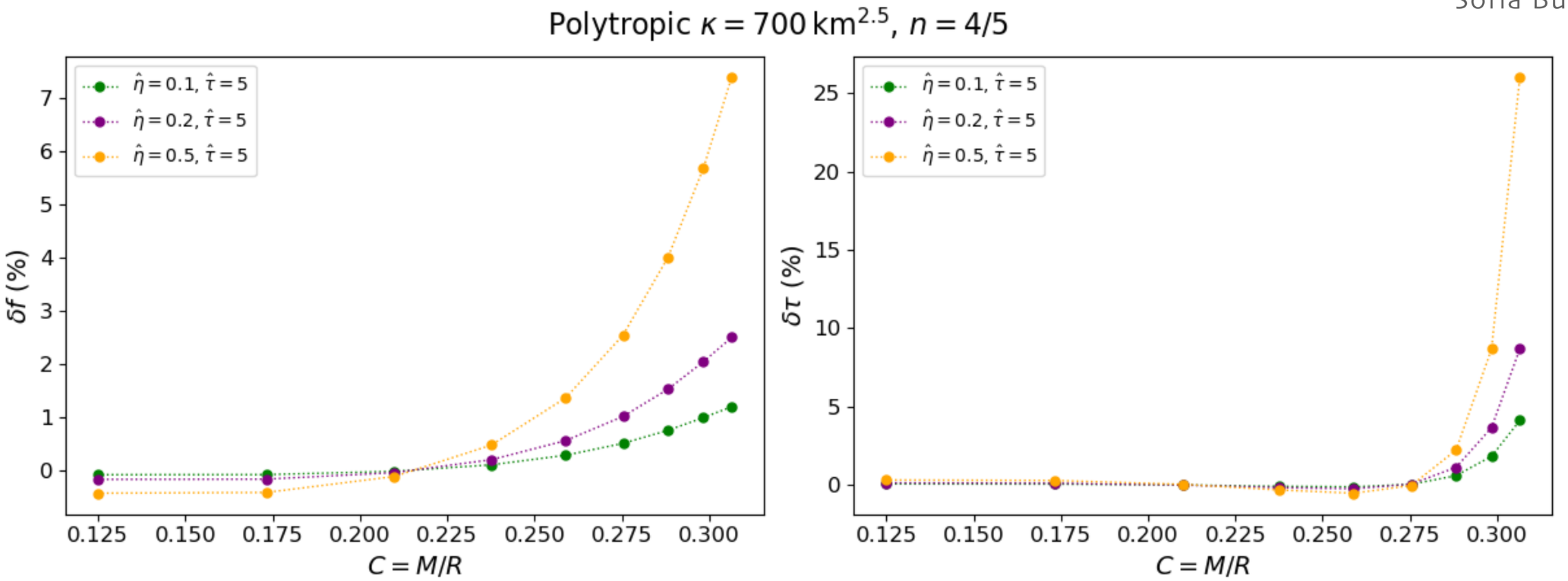


Inverse Cowling-approximation

w-modes are affected by viscosity !

Sofia Bussieres (UIB)

on-going: exact w-modes, generic (cold) EoS



Rotating viscous stars amplify (some) GWs

JRY & Cardoso **(2025)**

$$\bar{g}_{ab}dx^a dx^b = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2 - 2r^2\varpi \sin^2\theta dt d\phi$$

$$u = e^{-\nu/2}(\partial_t + \Omega\partial_\phi)$$

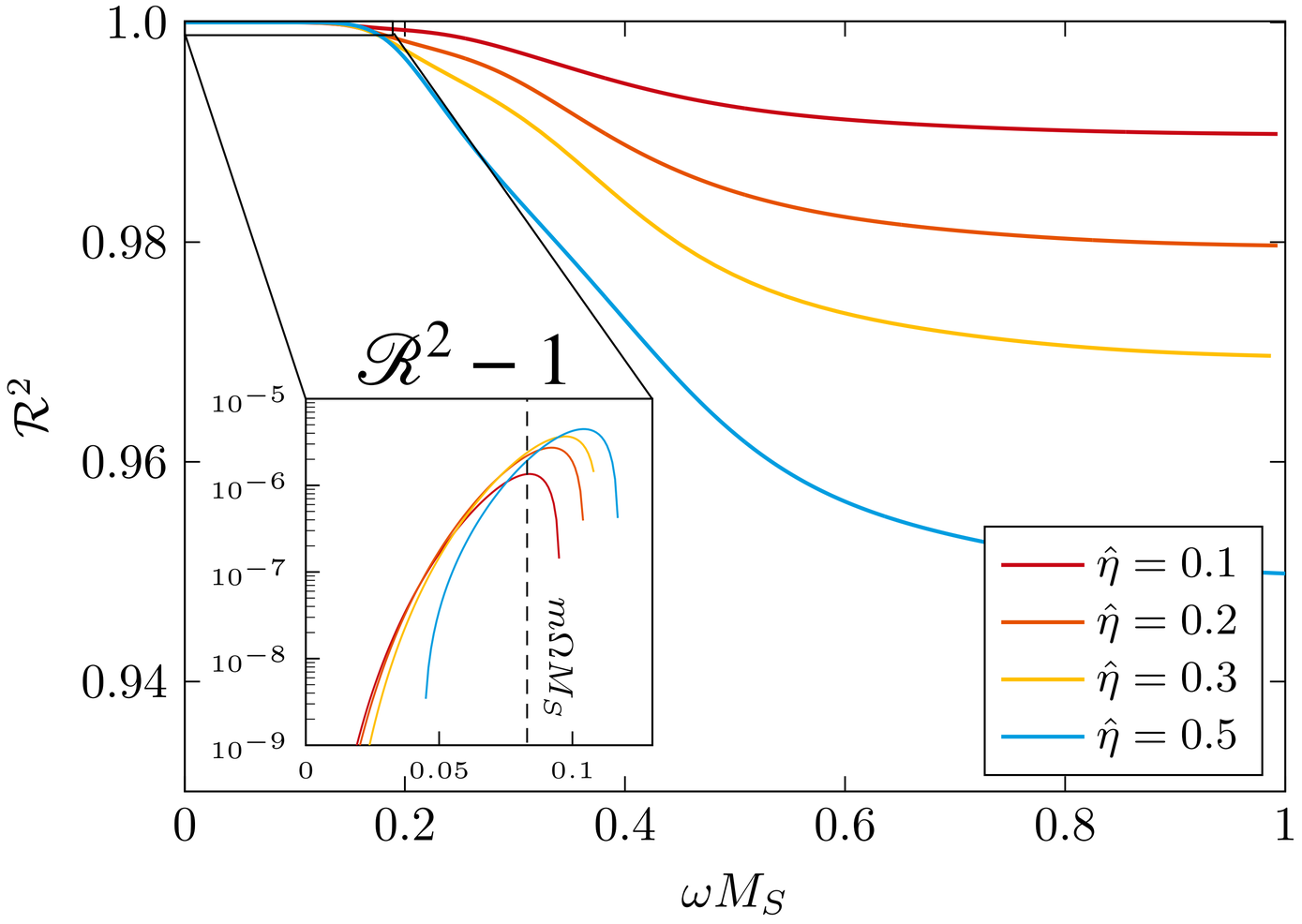
To linear order in Ω, we can treat the purely axial sector.

[w-modes] $$-\partial^2_{tt}\psi + \partial^2_{r_\star r_\star}\psi - V\psi = 16\pi\eta e^{\nu/2}(\partial_t\psi + A\beta) + \ldots$$

[η-modes] $$-\tau_Q\partial^2_{tt}\beta + \frac{\eta}{\varepsilon + p}\partial^2_{r_\star r_\star}\beta = L_1[\beta] + L_2[\psi] + \ldots$$

Rotating viscous stars amplify (some) GWs

JRY & Cardoso **(2025)**

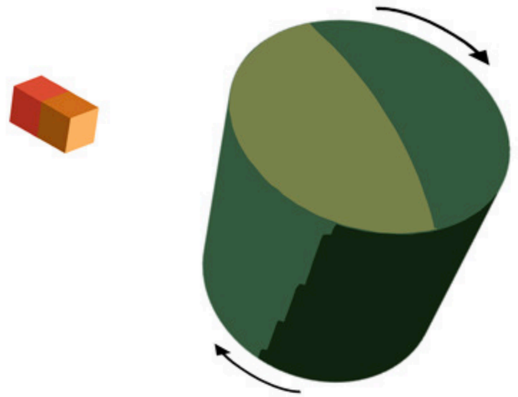


Why?

JRY & Cardoso **(2025)**

Let us throw a wave packet of frequency ω, angular momentum m, and power P towards the star

$$\dot{E} = ZPd\omega, \quad \dot{J} = Z\frac{m}{\omega}Pd\omega$$

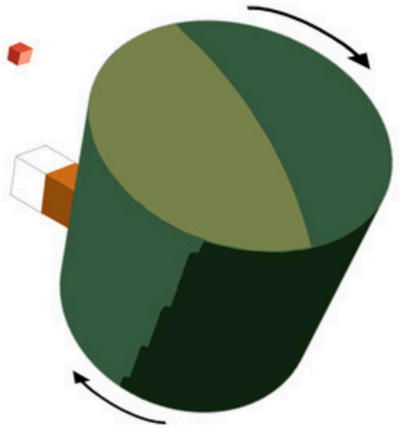


In the co-rotating frame

$$dE_0 = dE - \Omega dJ = dE(1 - m\Omega/\omega)$$

So the entropy increase is

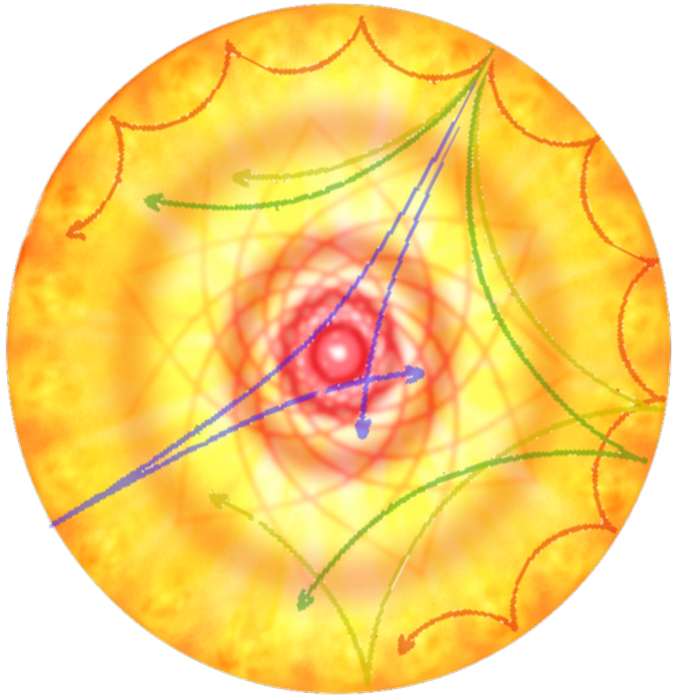
$$\dot{S} = \frac{\omega - m\Omega}{\omega T} ZPd\omega \geq 0$$



Brito, Cardoso, Pani (2015)

Why should I care?

JRY & Cardoso **(2025)**

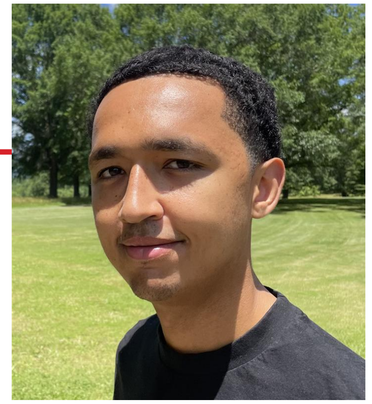


If there is a transition to a quark core: g-modes

f-mode: f~kHz, damping ~ 100ms
w-mode: f~ 10kHz, damping~0.01ms
g-mode: f~20Hz, damping~years

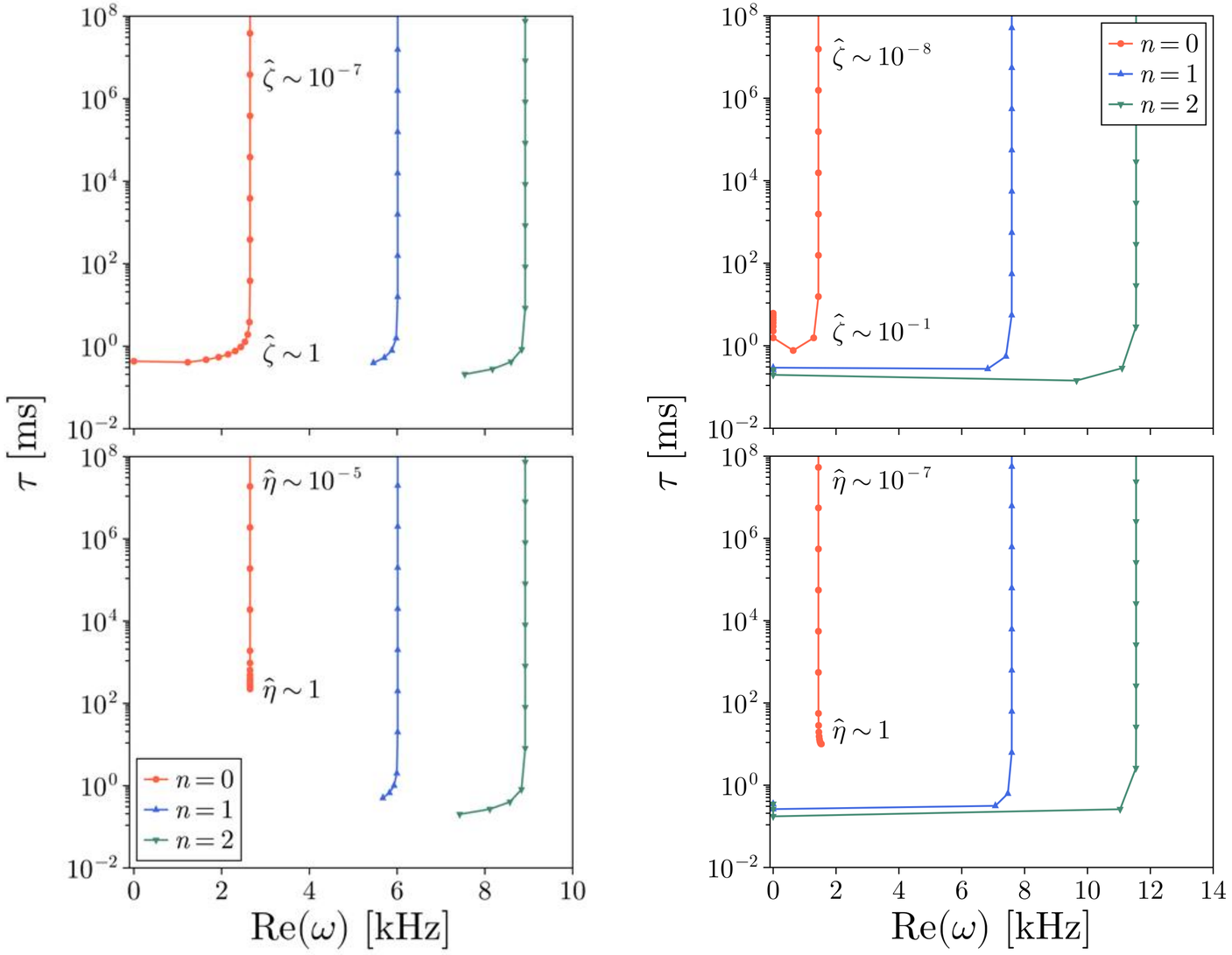
g-modes can be unstable: spin-down to $\Omega \lesssim f_G$

Radial Modes are damped by viscosity



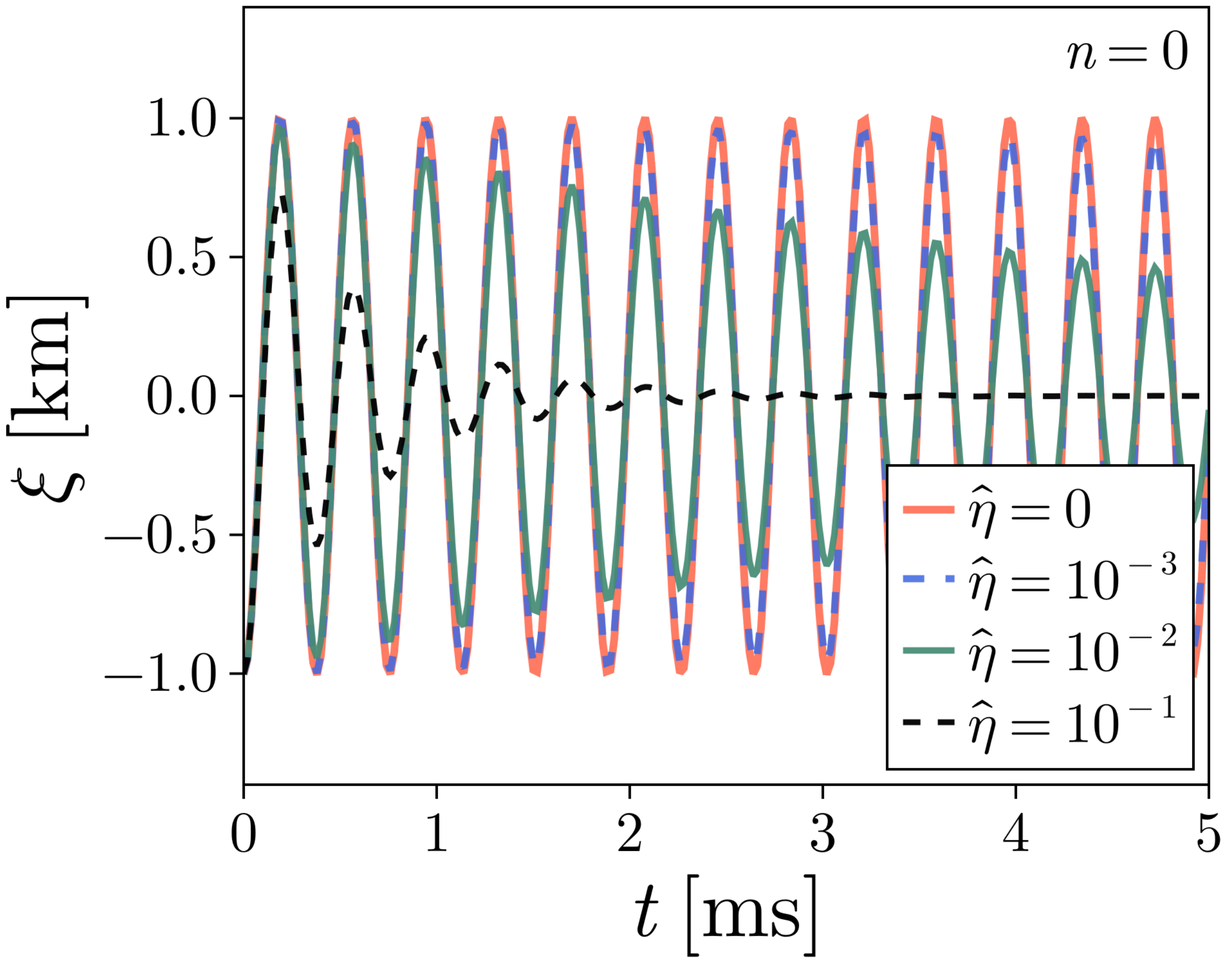
Lennox Keeble
(Cambridge)

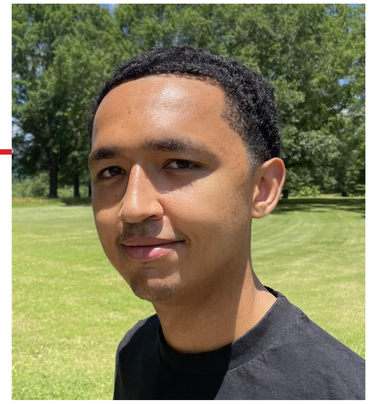
Radial perturbations in the Eckart frame



Radial Modes are damped by viscosity

Radial perturbations in the Eckart frame



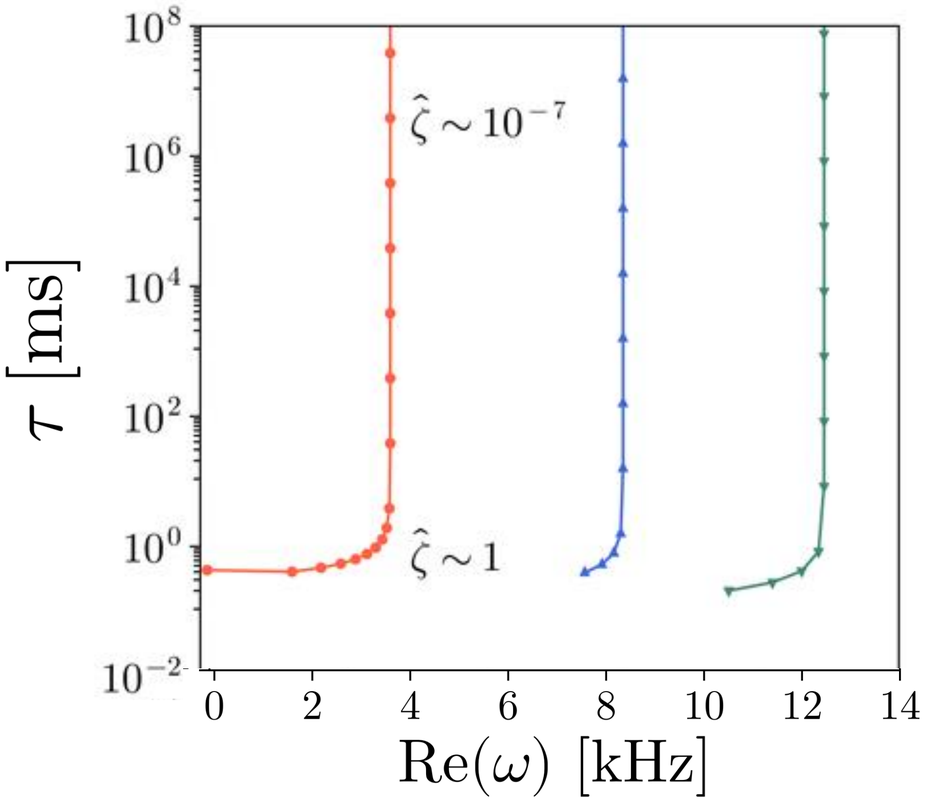


Lennox Keeble
(Cambridge)

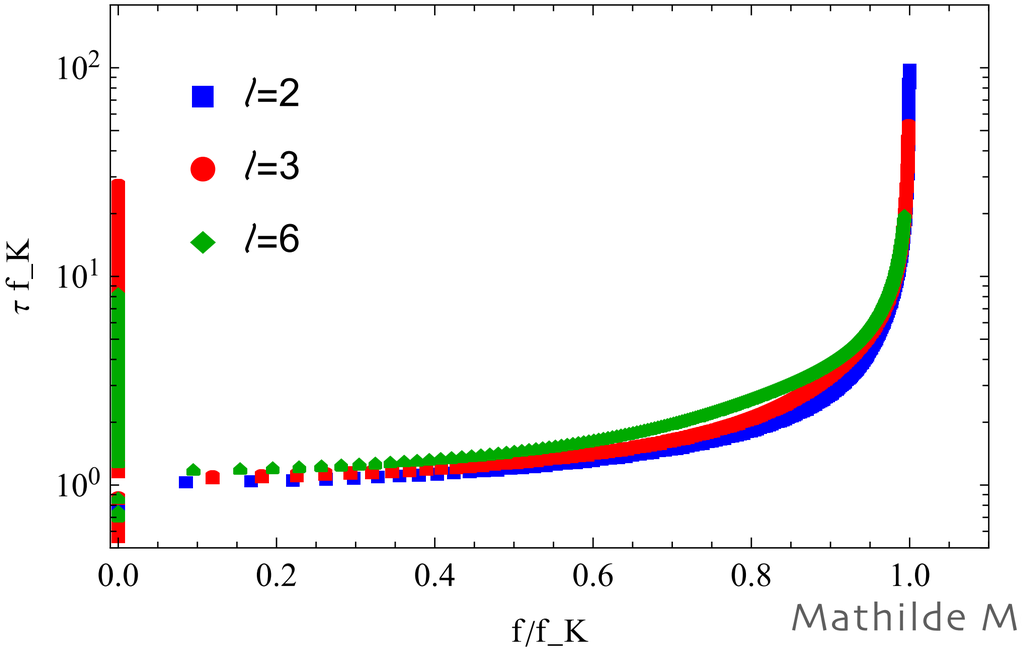
Remind you of something?

Lennox Keeble
(Cambridge)

Radial stellar modes:



Non-relativistic bubble modes:



Chandrasekhar (1958)

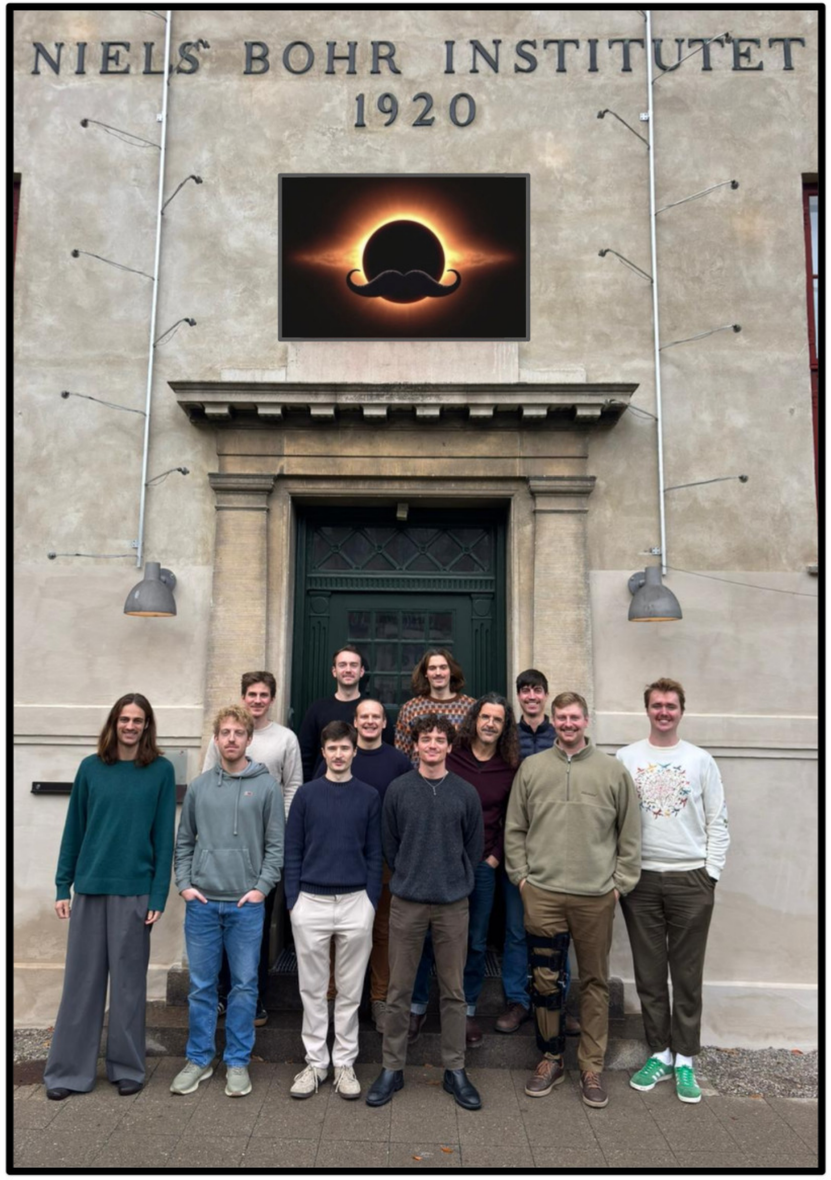
Mathilde Menu & Laura Micheletti
(BSc, AIX-UMarseille)

Coda

Aun dicen que el pescado es caro, J. Sorolla

Let's address the elephant in the room...

the moustache is only temporary!



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Take-aways

(i) Neutron Stars are a unique probe of dense matter

(ii) Hence, GWs are a unique probe of dense matter

(iii) Viscous effects distinguish hyperons / quarks

(iv) BDNK is a good theory to include viscous effects

(v) We can now study perturbations of BDNK stars

(vi) Viscous stars absorb high-frequency GWs (tidal heating)

(vii) Viscous stars are superradiant: g-mode instabilities?

(viii) Asteroseismology may constrain transport coefficients!!!

[ask me about black hole ringdown & nonlinearities]