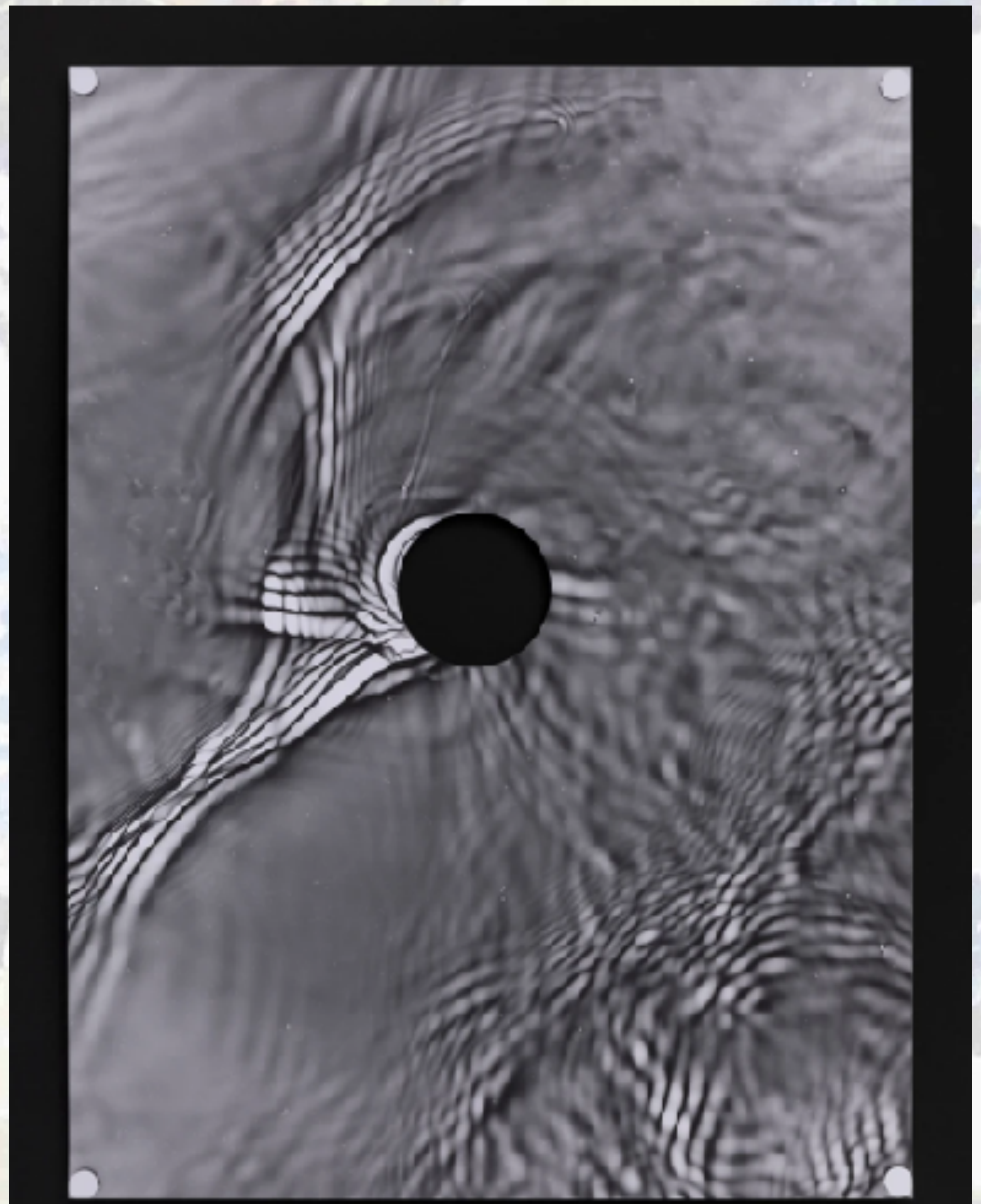
Turbulence of gravitational waves



Reservoir (2023) Sjoerd Knibbeler

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THE CENTER OF GRAVITY

KØBENHAVNS UNIVERSITET

Danmarks Grundforskningsfond
Danish National Research Foundation

Set list

Heat Waves [Glass Animals]

Ohms [Deftones]

Hurricane [Scorpions]

Highway star [Deep Purple]

Here comes the Sun [The Beatles]

Heat waves

Electromagnetic Waves

No charges, flat space, Lorenz gauge

$$F_{ab} = \partial_{[a} A_{b]}$$ *Vector Potential*

$$\partial^b F_{ab} = 0$$ *Maxwell Eqs*

$$\partial^b \partial_a A_b - \partial^b \partial_b A_a = 0$$

$$\Box A_a = \partial_a \partial^b A_b = 0$$ *Lorenz gauge!*

$$\Box = -c^{-2} \partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2$$

Fluid Waves

Perfect fluids are already non-linear

$$\partial_t \vec{u} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} = -\rho^{-1}\vec{\nabla}p$$

However, small fluctuations satisfy a wave equation

$$\vec{u} = \varepsilon \vec{u}' \,, \quad p = p_0 + \varepsilon p' \,, \quad \rho = \rho_0 + \varepsilon \rho'$$

$$\partial_t^2 \vec{u}' = -\rho_0^{-1} \vec{\nabla} \partial_t p' \,, \quad \partial_t \rho' = -\rho_0 \vec{\nabla} \vec{u}'$$

$$-\partial_t^2 \vec{u}' + c_s^2 (\vec{\nabla} \cdot \vec{\nabla}) \vec{u}' = 0$$

Irrotational flow!

Gravitational Waves

Einstein's equations are also non-linear

$$\partial^c \partial_c g_{ab} = -\partial_b g^{cd} \partial_c g_{ad} - \partial_a g^{cd} \partial_c g_{db}$$

Harmonic gauge

Pretorius '05

Their fluctuations satisfy a wave equation as well

$$g_{ab} = \eta_{ab} + \varepsilon h_{ab} \qquad \Box h_{ab} = 0$$

But we can also write a non-linear wave equation!

$$\Box C_{abcd} + C_{ab}^{\ \ ef} C_{efcd} + 4 C^{e\ f}_{\ a\ [d} C_{c]feb} = 0$$

Penrose '60

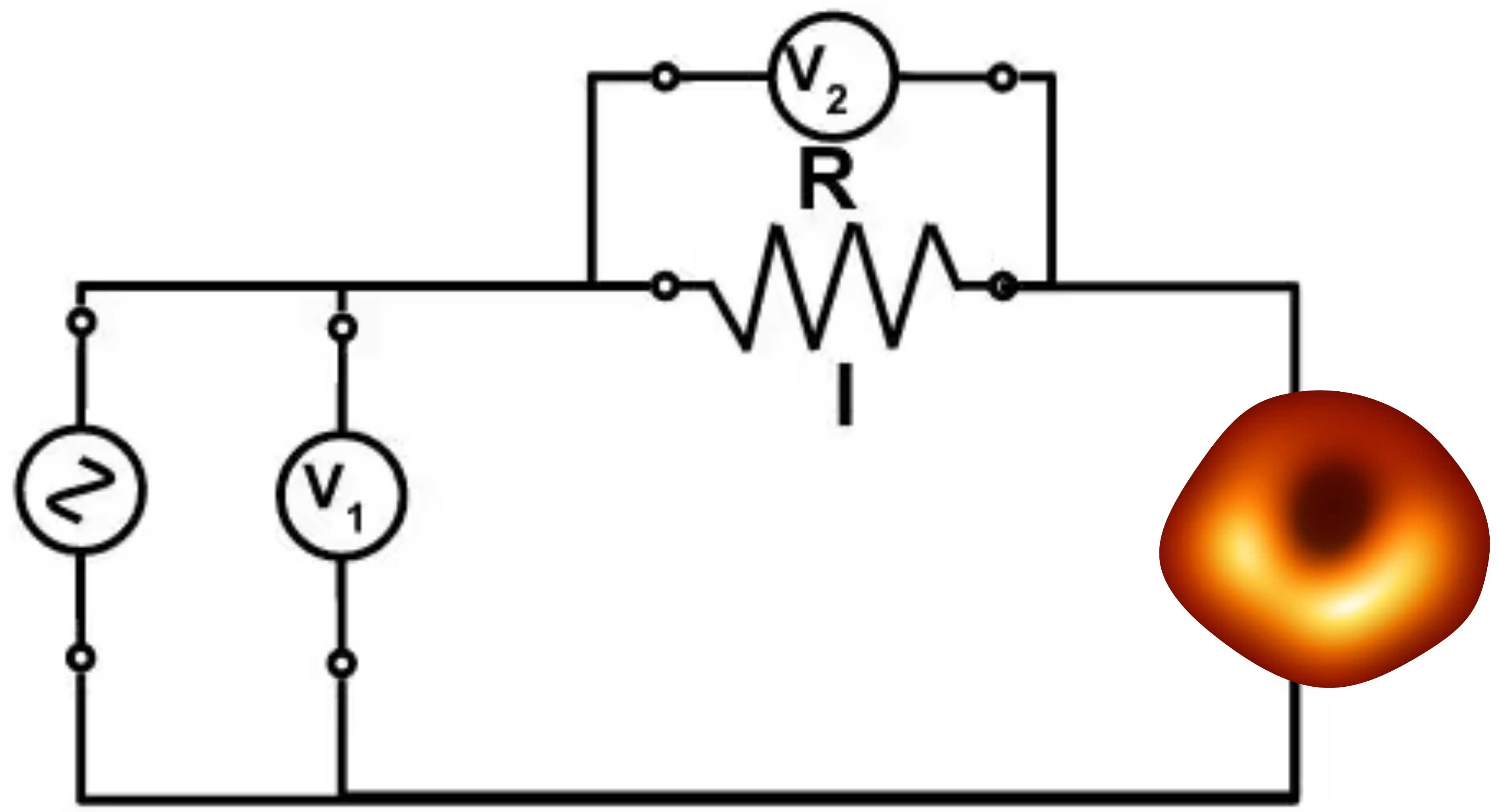
Stewart & Walker '73

Comparison

EM	GR	Hydro
$\Delta_{\mathrm{dR}} F = 0$	$\Delta_{\mathrm{dR}} C = 0$	???
Linear	Nonlinear	Nonlinear
$\Box A = 0$	$\Box h = 0$	$\Box u' = 0$

Ohms

What's the impedance of a black hole?



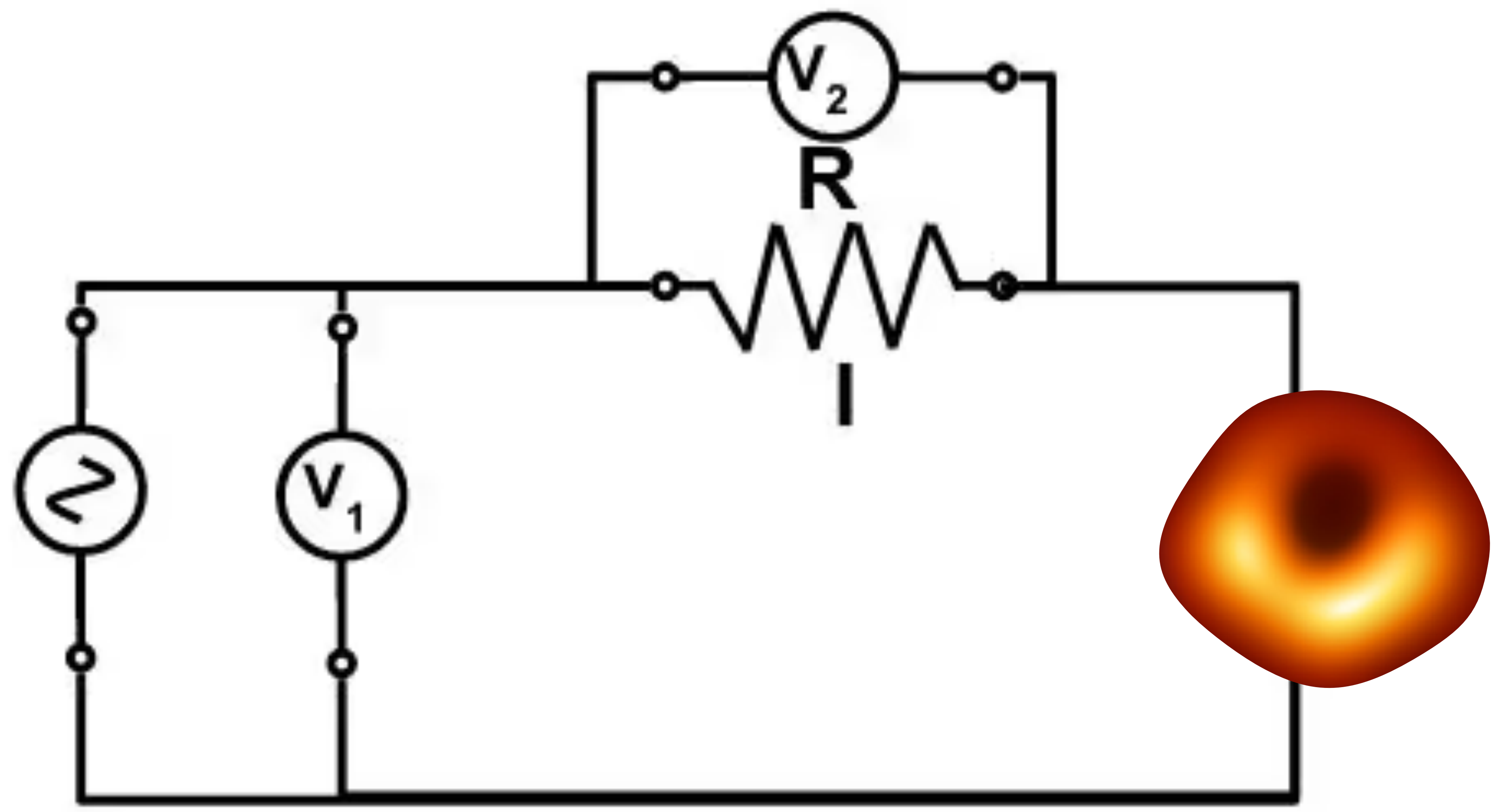
A) 0 Ohms

B) 377 Ohms

C) Infinity

D) It depends!

What's the impedance of a black hole?



A) 0 Ohms

B) 377 Ohms

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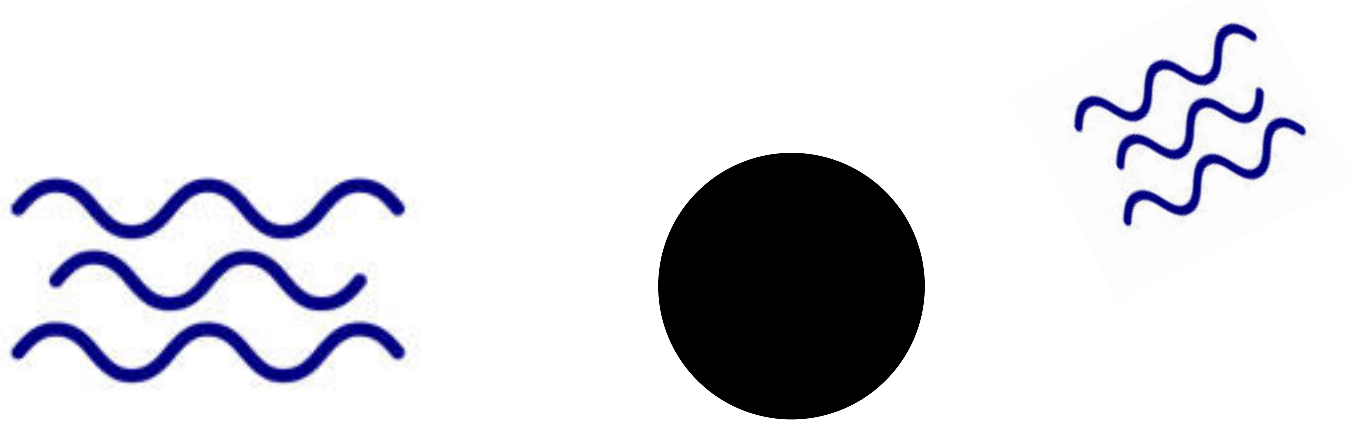
D) It depends!

Damour '78
Thorne+ '86

The membrane paradigm

Viscous fluid membrane

$$\eta = \frac{1}{16\pi}\,, \qquad \zeta = -\frac{1}{16\pi}$$



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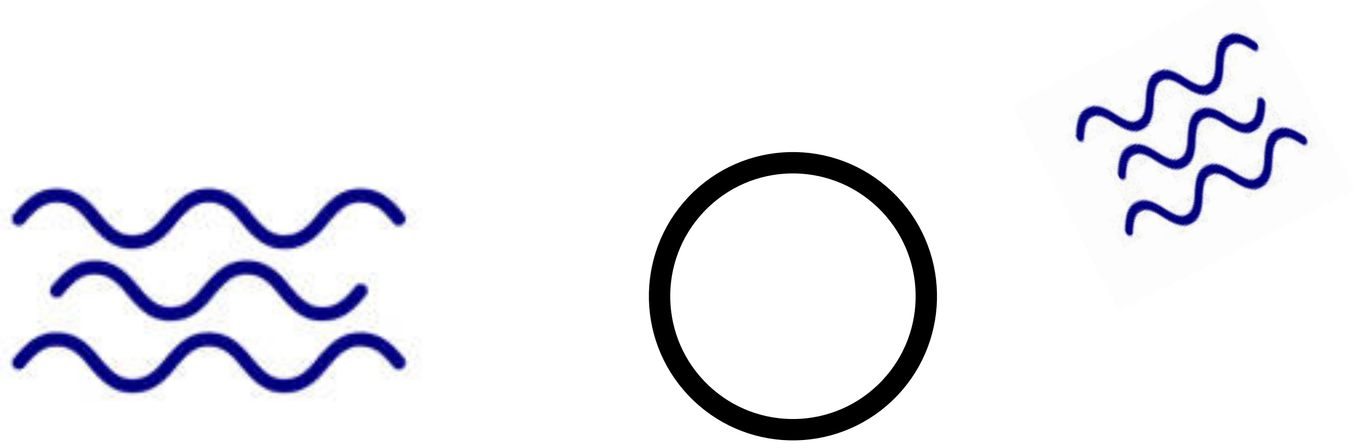


$$G_{vv} = 0$$

$$G_{vA} = 0$$

$$u^A \nabla^B T_{AB} = 0$$

$$\perp^{AC} \nabla^B T_{AB} = 0$$

Damour '78

Thorne+ '86

The membrane paradigm

Gravity in 3+1 dimensions = Viscous hydro in 2+1 dimensions

(Not quite, ask me later!)

Donnay & Marteau '19
***RY** & Lehner '22*
Freidel & Jai-akson '22,'24

Hurricane

Fujiwhara effect

Radius (hurricane) > Height (atmosphere)

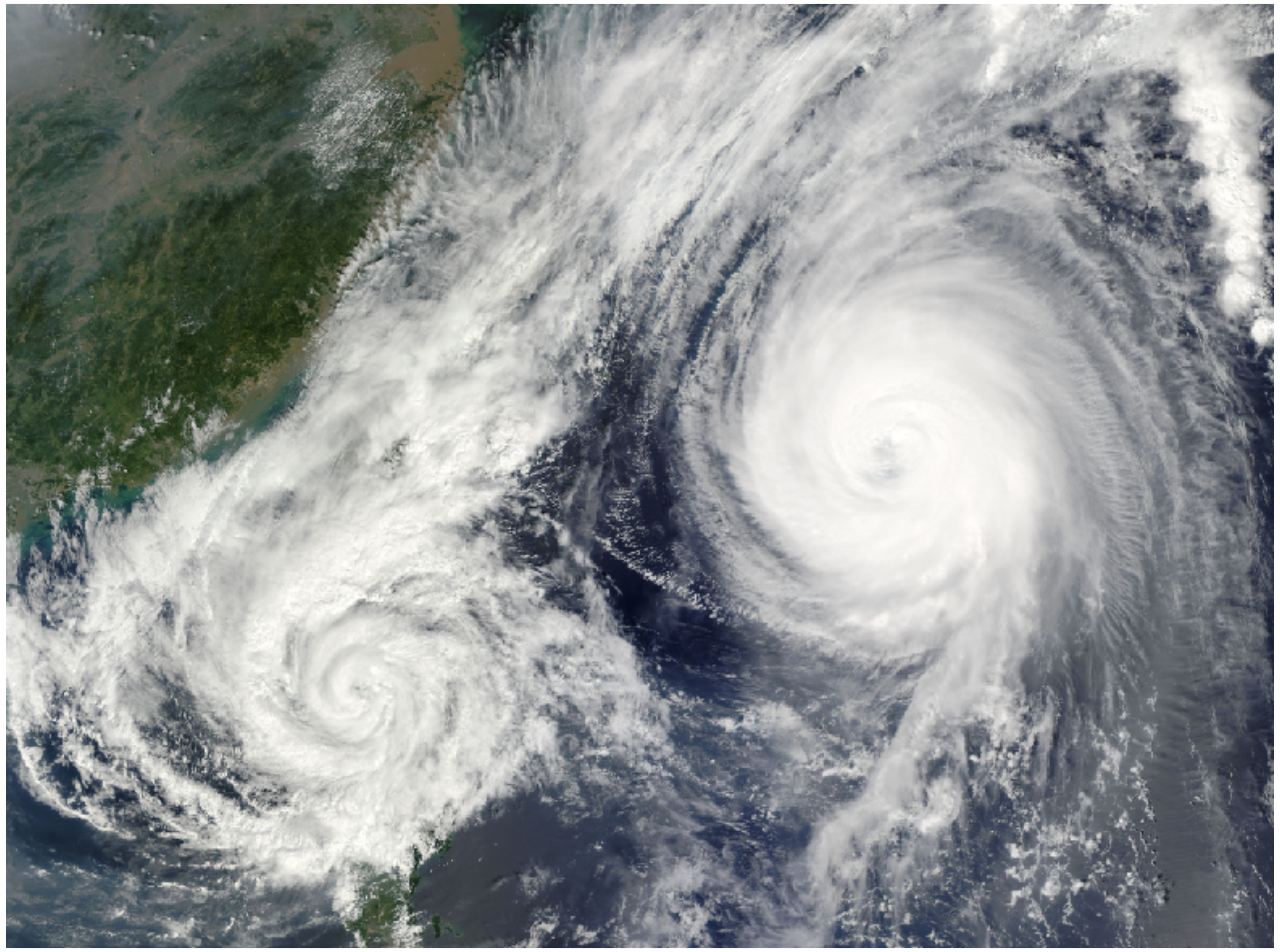
Enstrophy

$$\Omega = \int d^2x \, |\nabla u|^2$$

$$\partial_t \Omega = -\nu \int d^2x |\nabla \omega|^2$$

Inverse cascade UV —>IR

(Vortex merger is favourable)



Holography 101

Quantum Gravity in (D+1) AdS = D-dim CFT

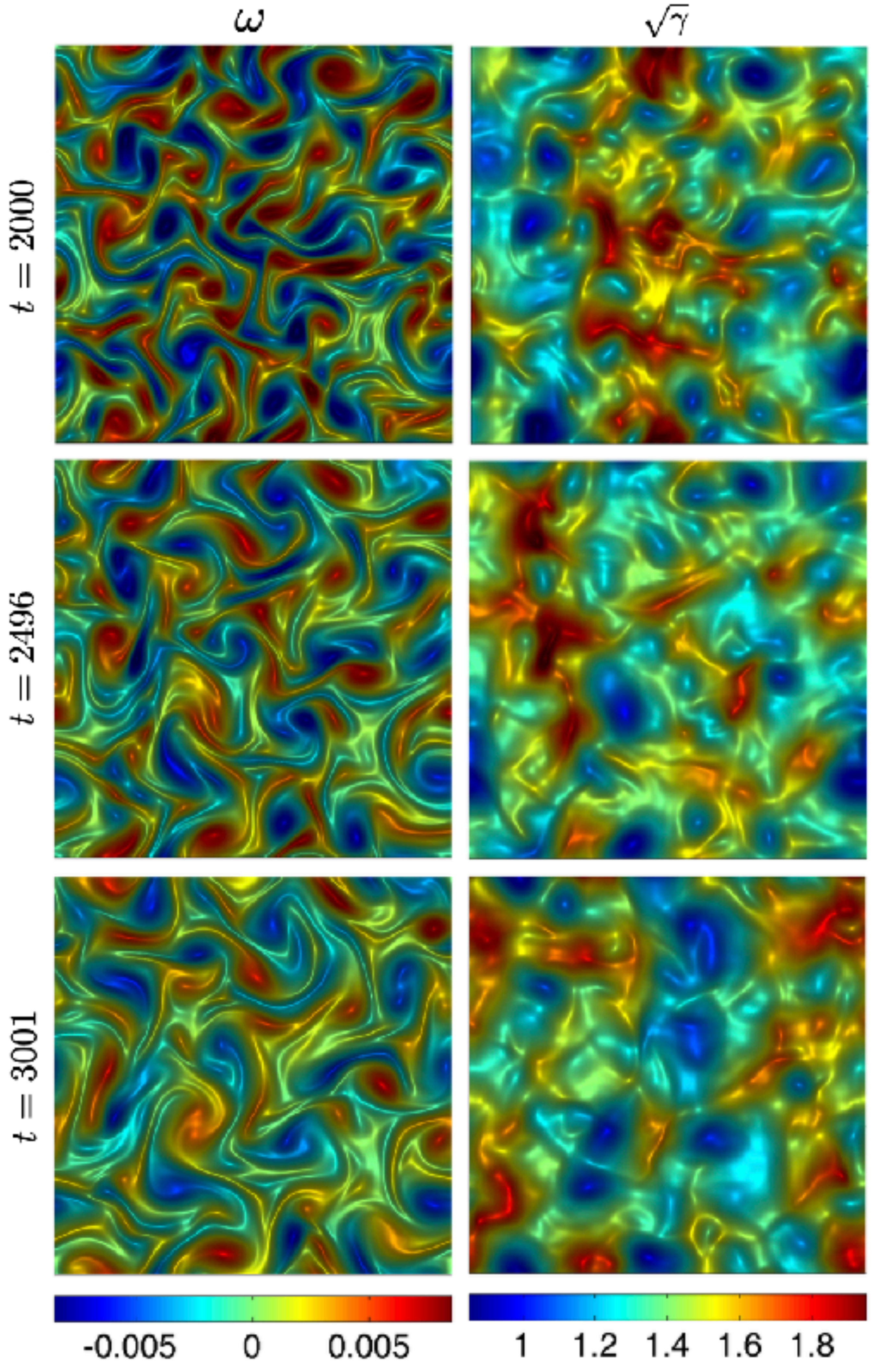
A simpler version, which does not require string theory

GR in (D+1) AdS = D-dim relativistic conformal fluid

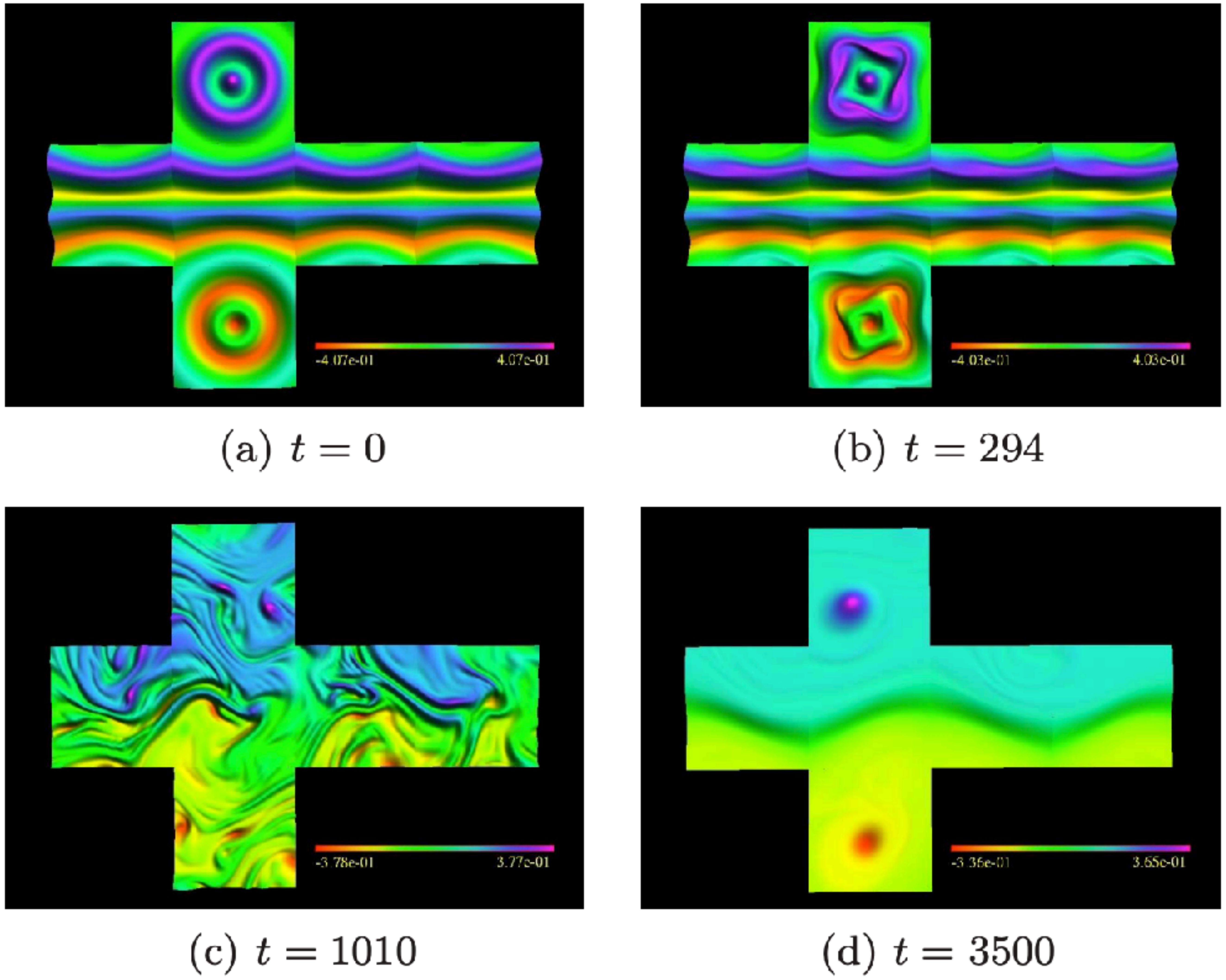
As a consequence…

Turbulent (D+1) Black Hole in AdS = D-dim turbulent rel conf fluid

Holography 101



Adams+ '12


(a) $t = 0$
(b) $t = 294$
(c) $t = 1010$
(d) $t = 3500$

Carrasco+ '12

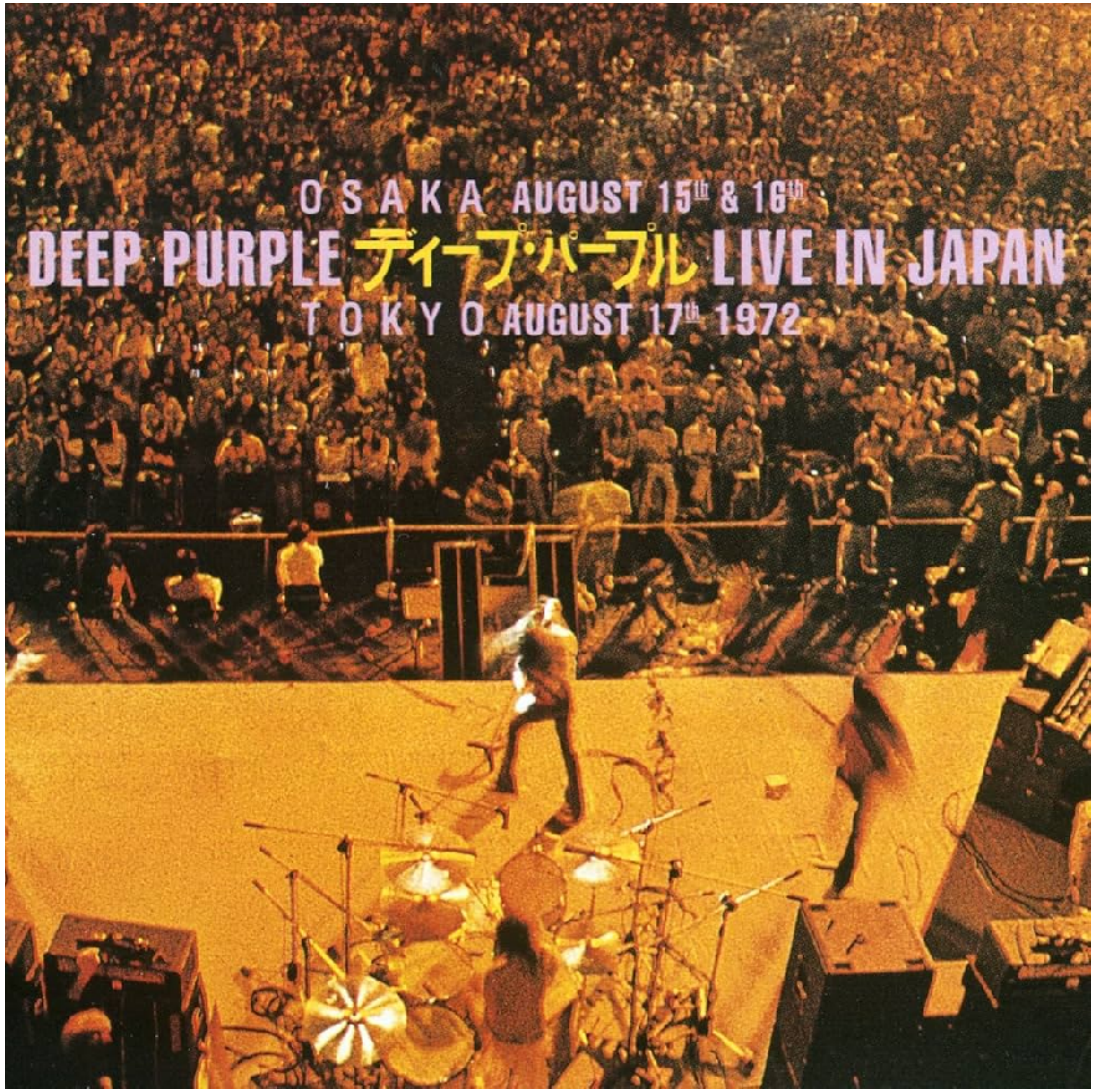
Take-aways

In some regimes (AdS, conformal fluids), GR=Hydro

Is it surprising?

On-going work related to fluid/gravity
correspondence outside of AdS

Highway Star



Nonlinearities!

1+1d elastic string

$$\ddot{\xi} - c_L^2 \xi'' - \frac{1}{\tau_L}\dot{\xi} = (c_L^2 - c_T^2)\eta' \Delta\,,$$

$$\ddot{\eta} - c_L^2 \eta'' - \frac{1}{\tau_T}\dot{\eta} = (c_T^2 - c_L^2)(1+\xi')\Delta\,,$$

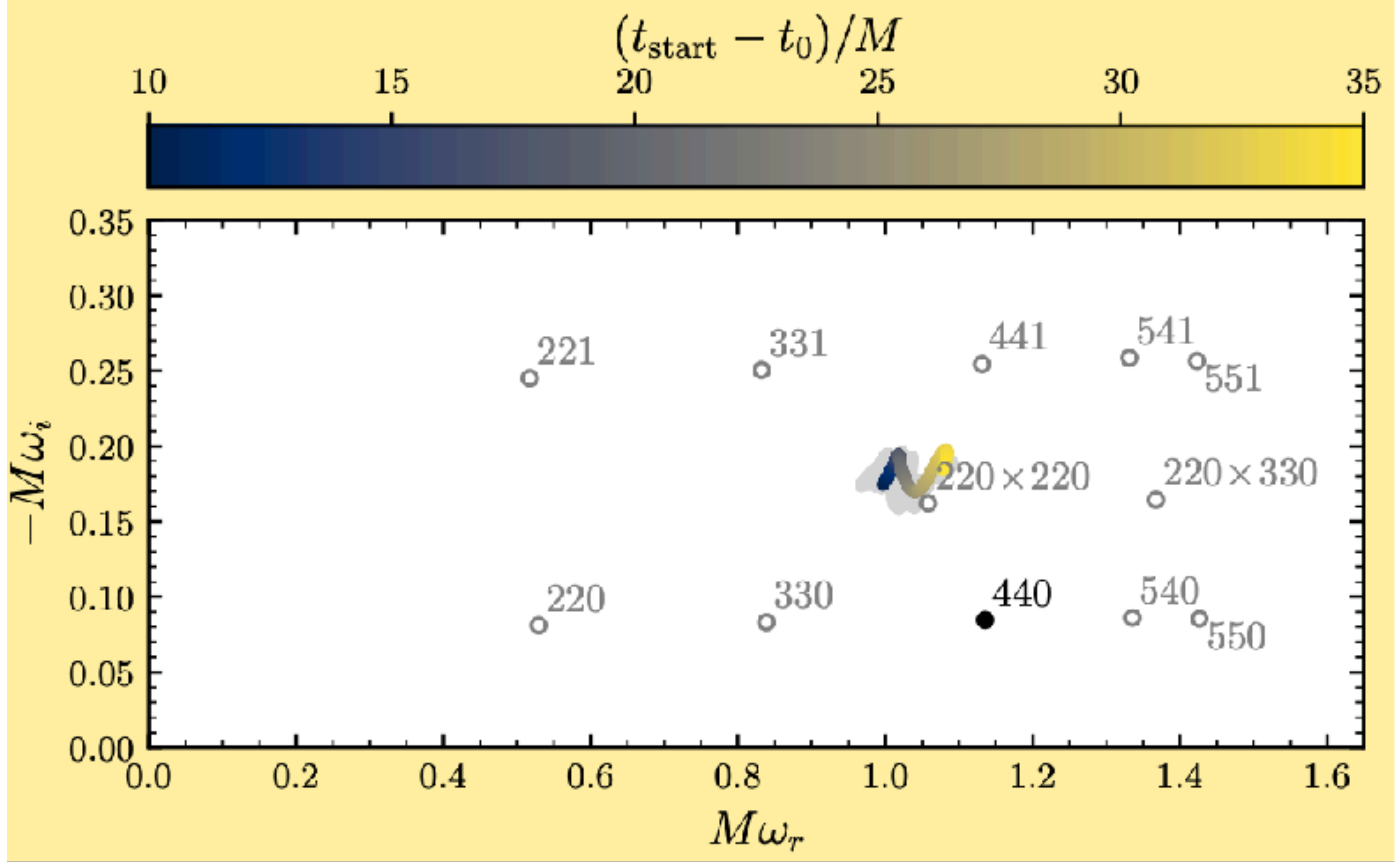
$$\Delta = \frac{(1+\xi')\eta'' - \eta'\xi''}{\left[(\eta')^2 + (1+\xi')^2\right]^{3/2}}$$

Large perturbations excite higher harmonics & L-T couplings

(real) Black Holes

Campanelli & Lousto '98

$$\mathcal{O}\psi^{(1)} = 0$$
$$\mathcal{O}\psi^{(2)} = \mathcal{S}[\psi^{(1)}, \psi^{(1)}]$$

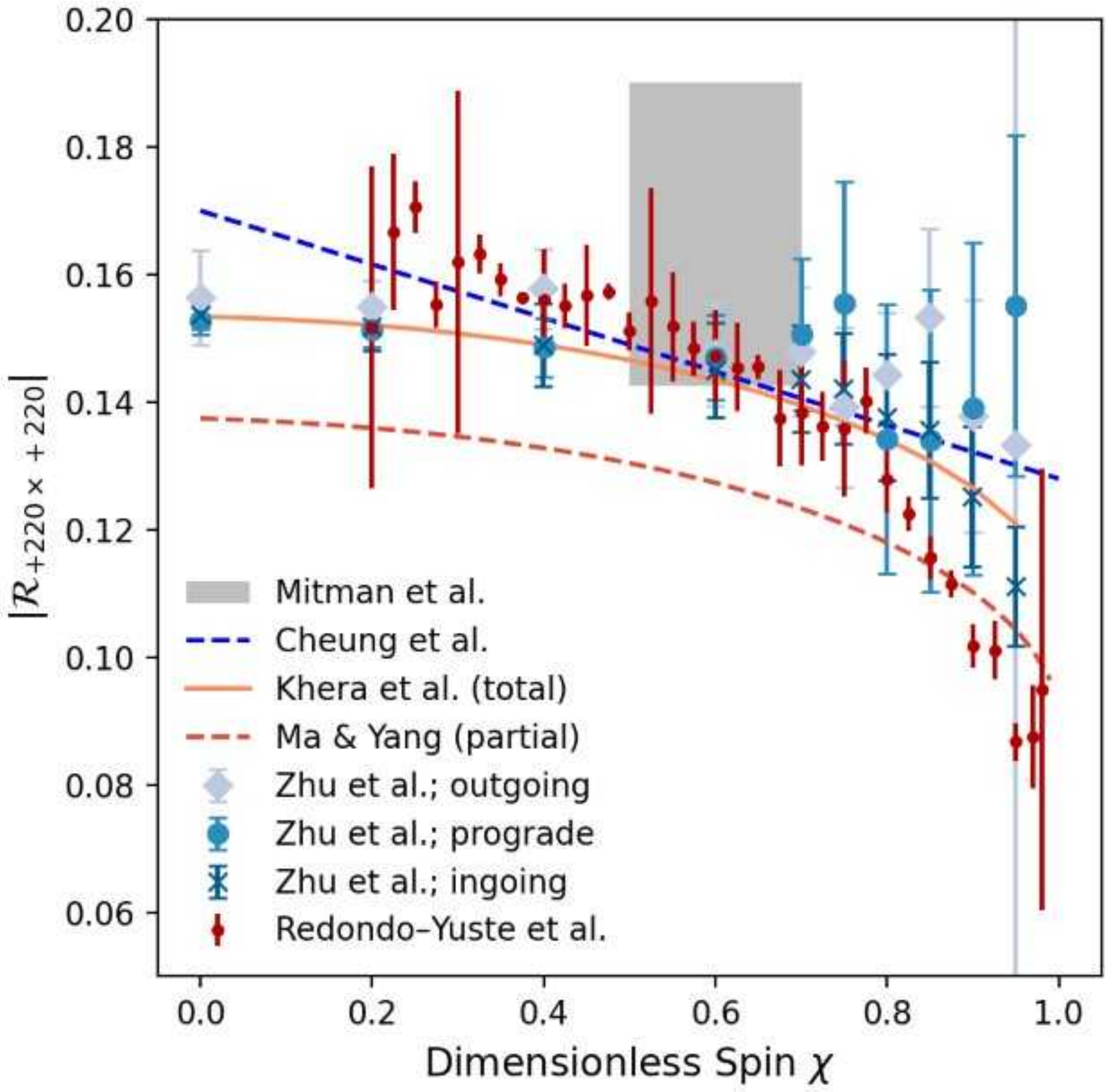


Observable in LISA

Cheung+ '22
Mitman+ '22

(real) Black Holes

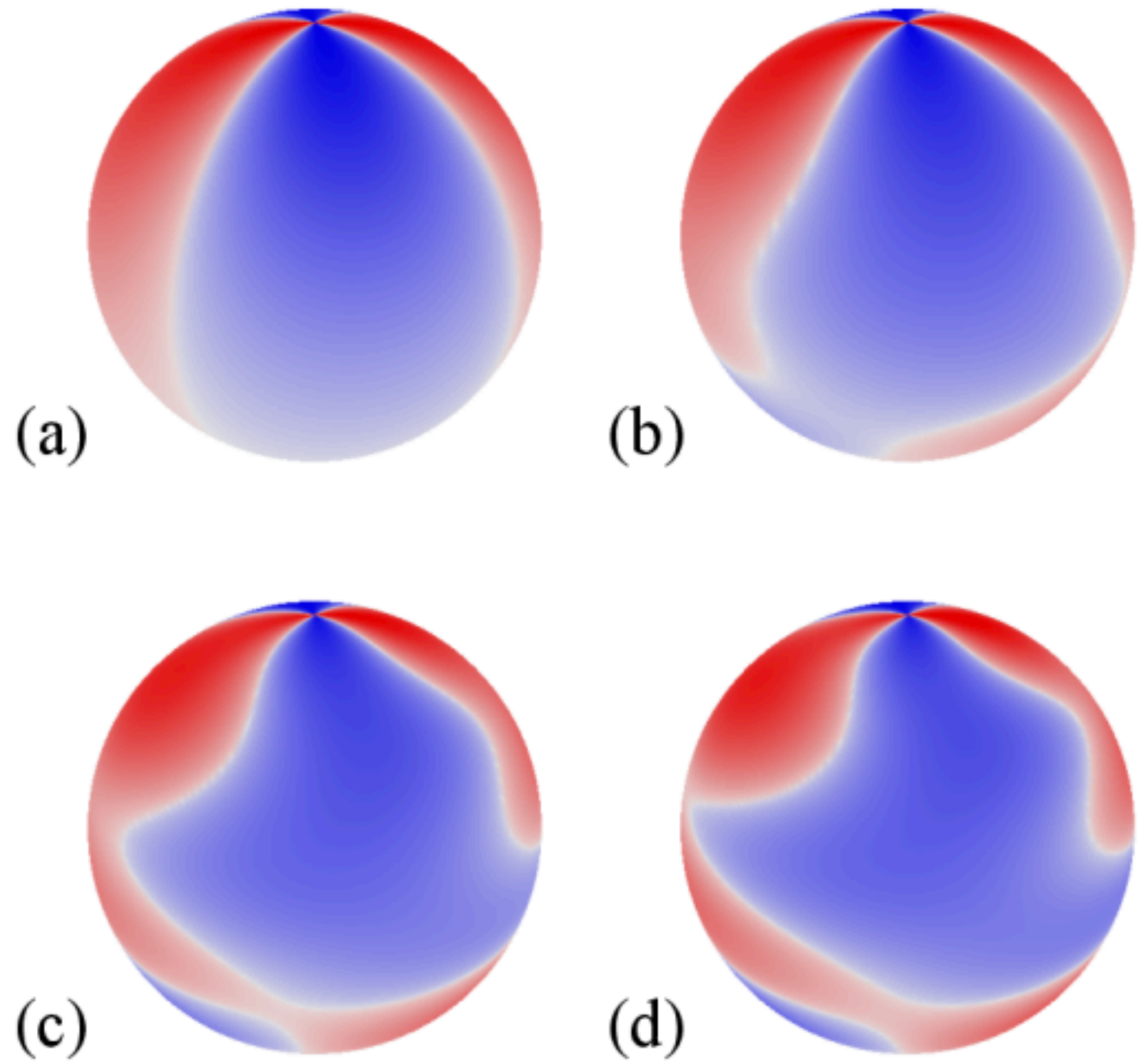
The coupling is small!



(real) Black Holes

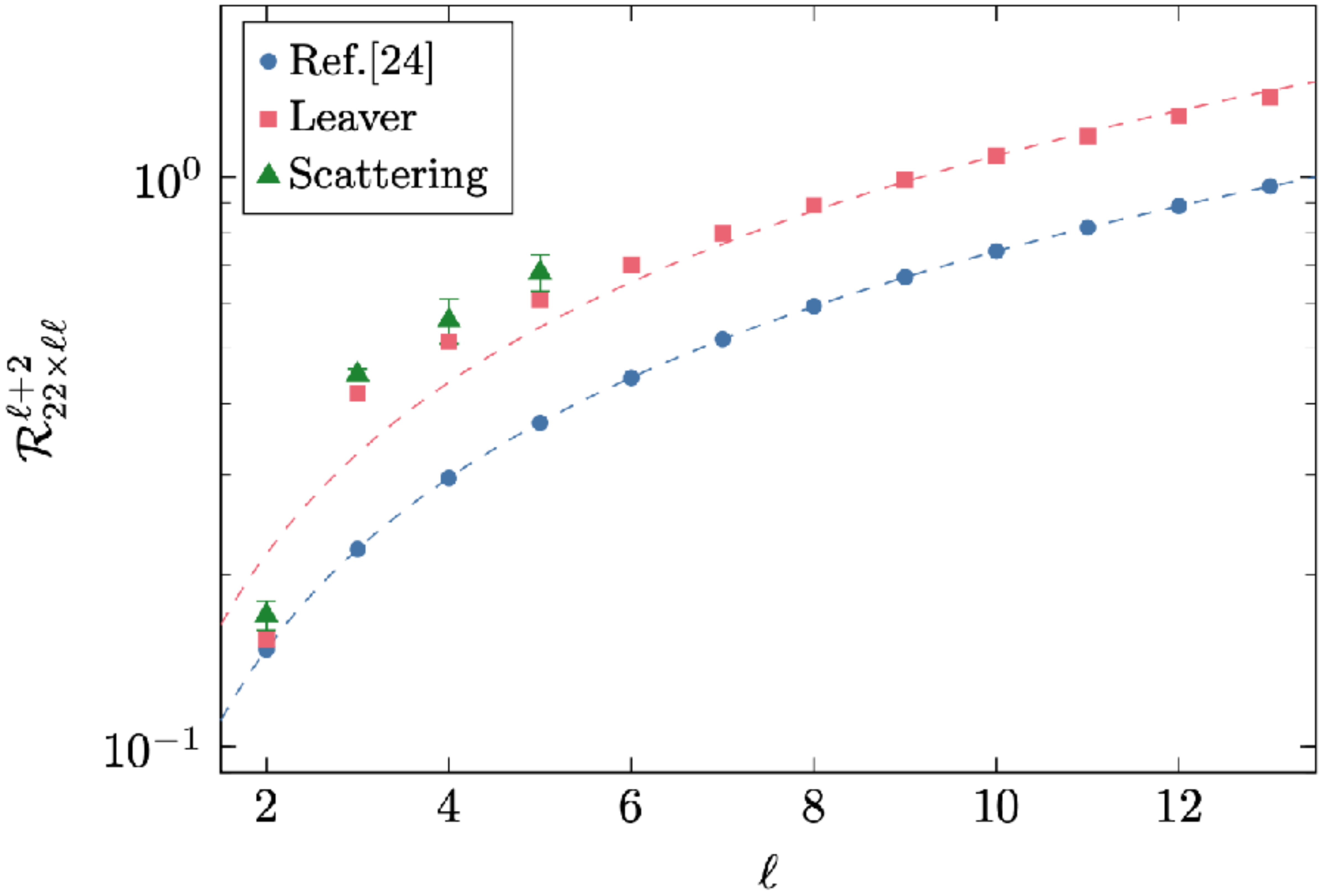
Perhaps not always…

High spins



Yang+ '14

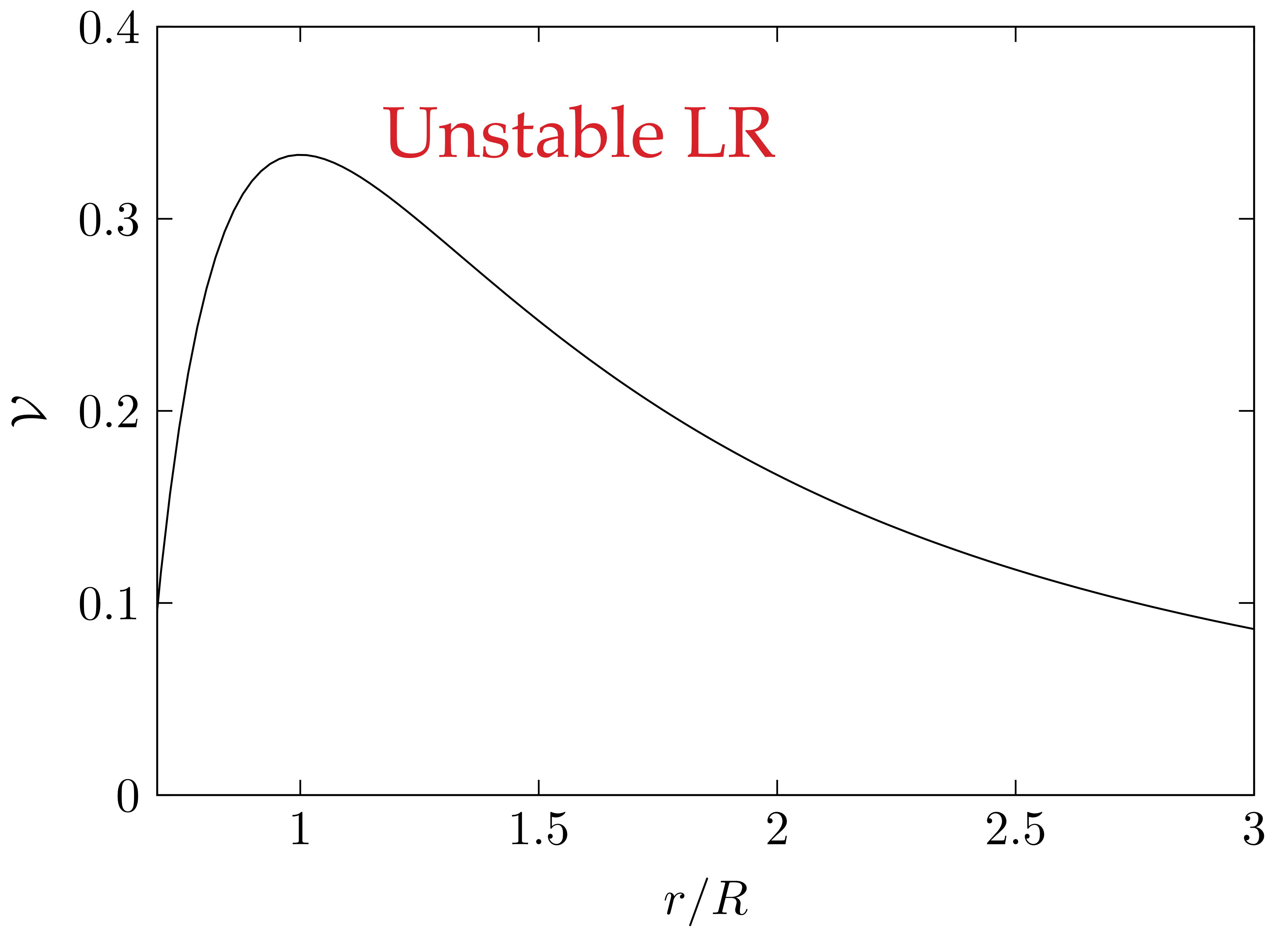
High frequency modes

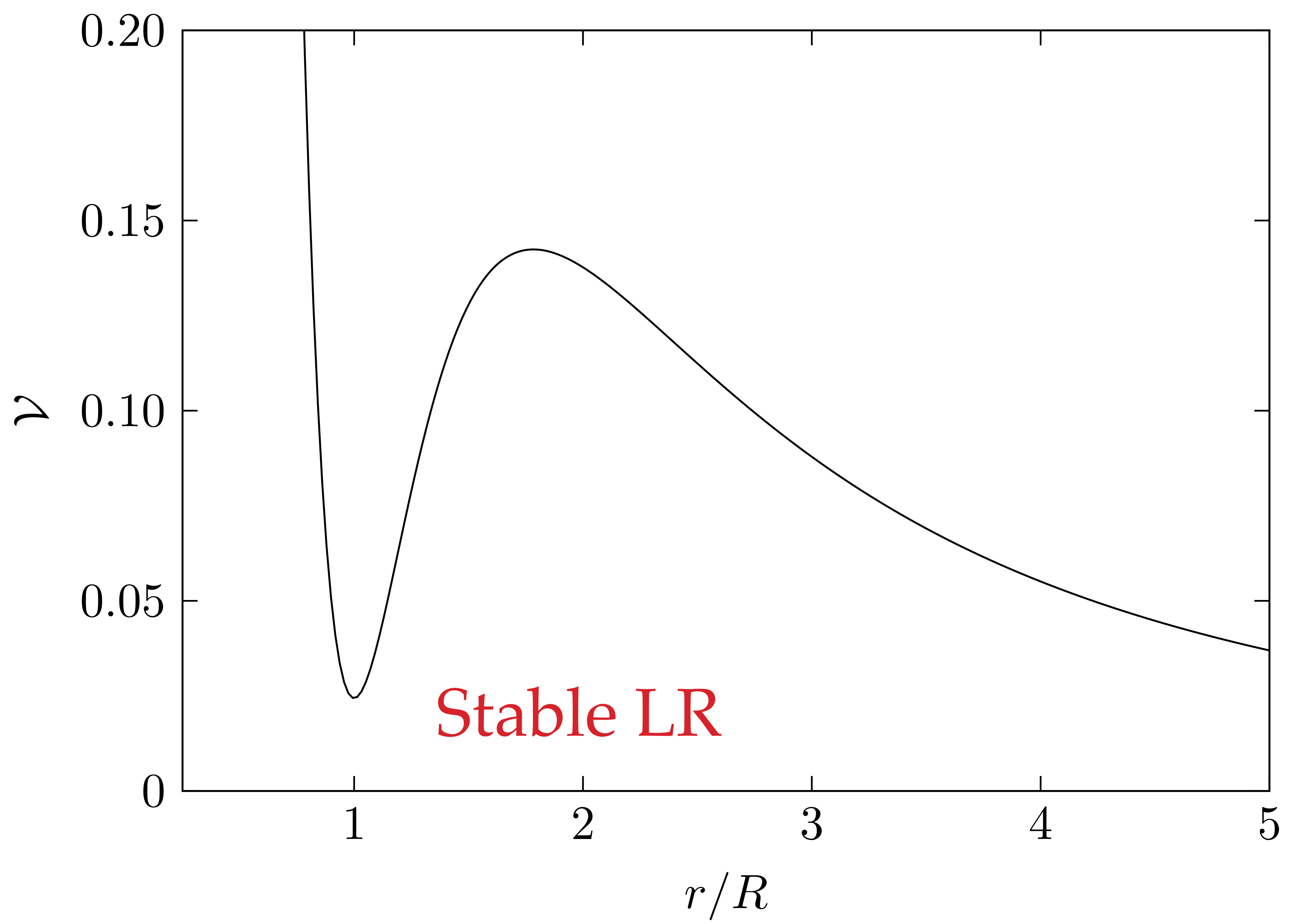


*Buccioti+ (**RY**) '25*

(Not so real) Black Holes

What if compact objects don't have horizons?



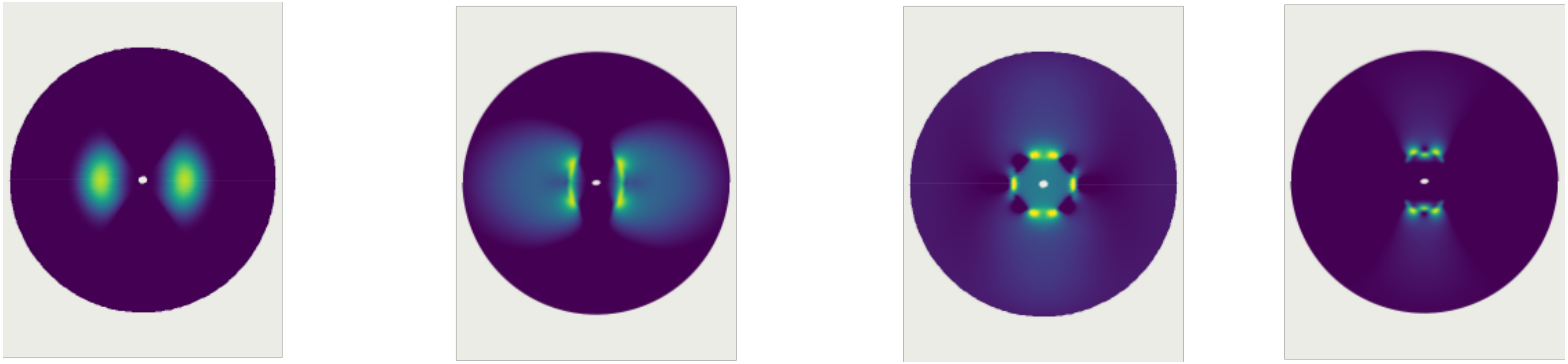


Trapping —> instability?

Keir '14, Cardoso+ '14

(Not so real) Black Holes

$$\Box\Phi = \Phi^3$$

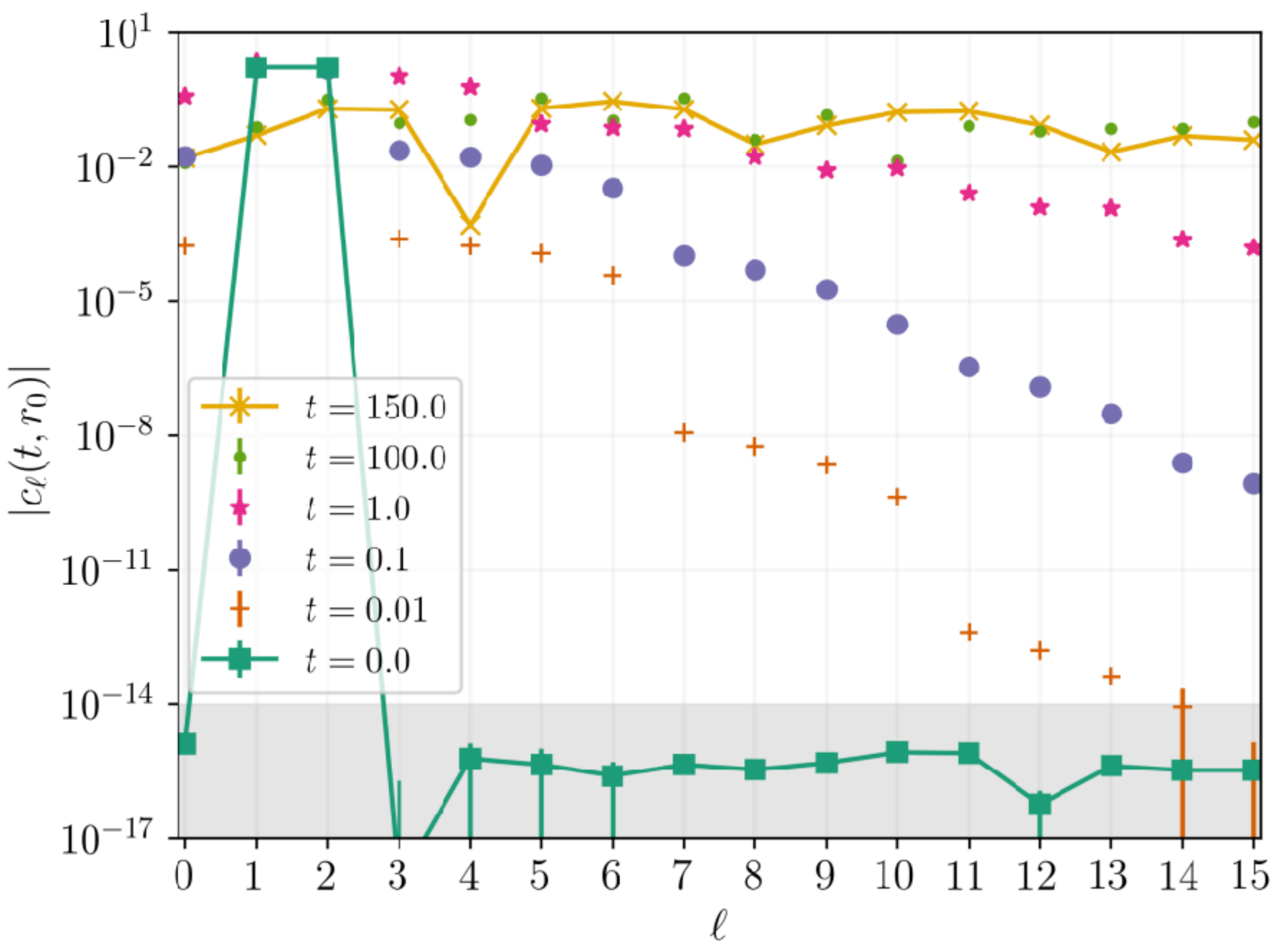


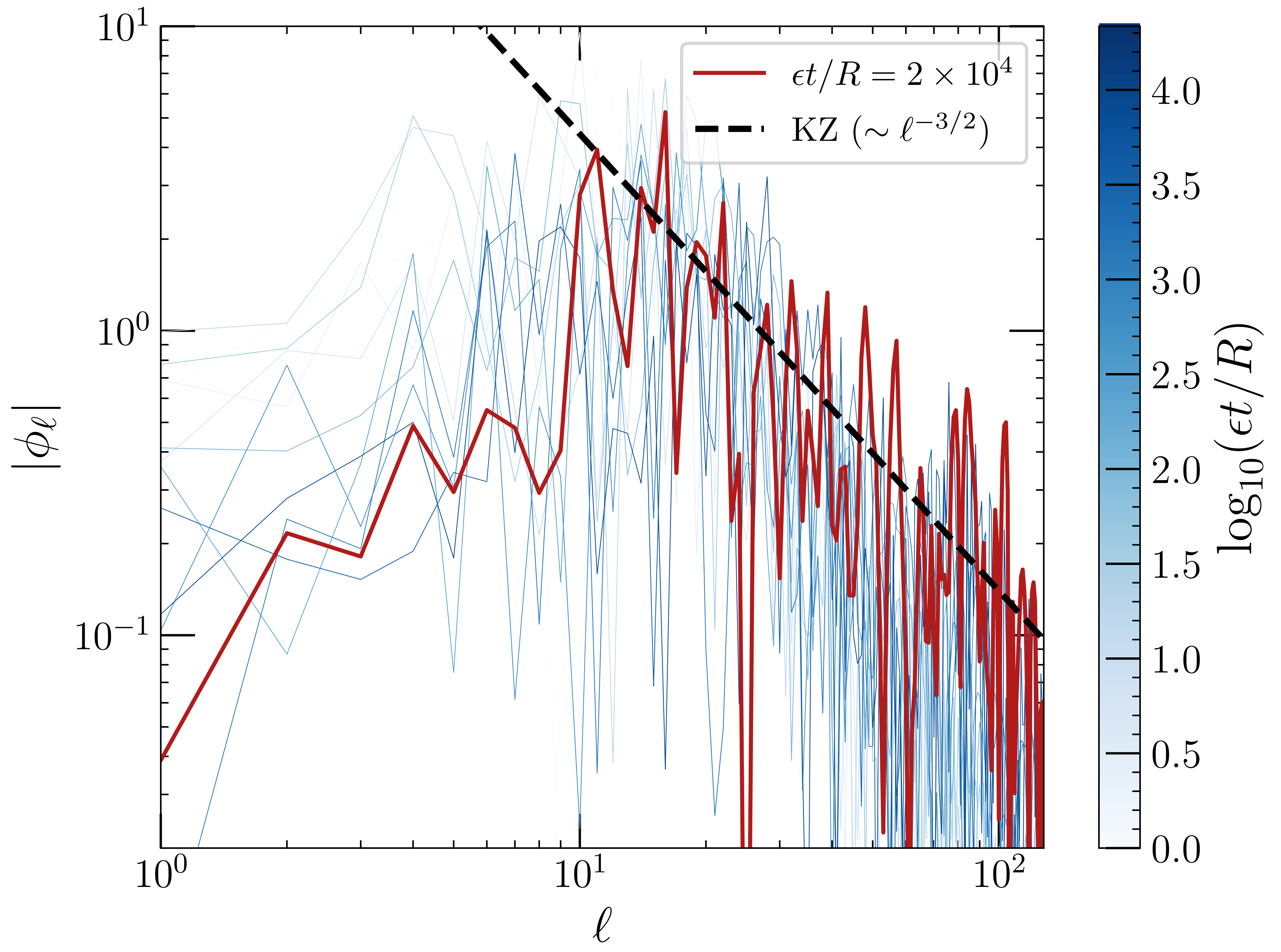
Benomio+ '24

***RY** & Cárdenas-Avendaño '25*

(Not so real) Black Holes

Radiation is trapped, and leads to a direct energy cascade





Take-aways

GR is nonlinear, and we can see clear nonlinear effects

GR is also very dispersive

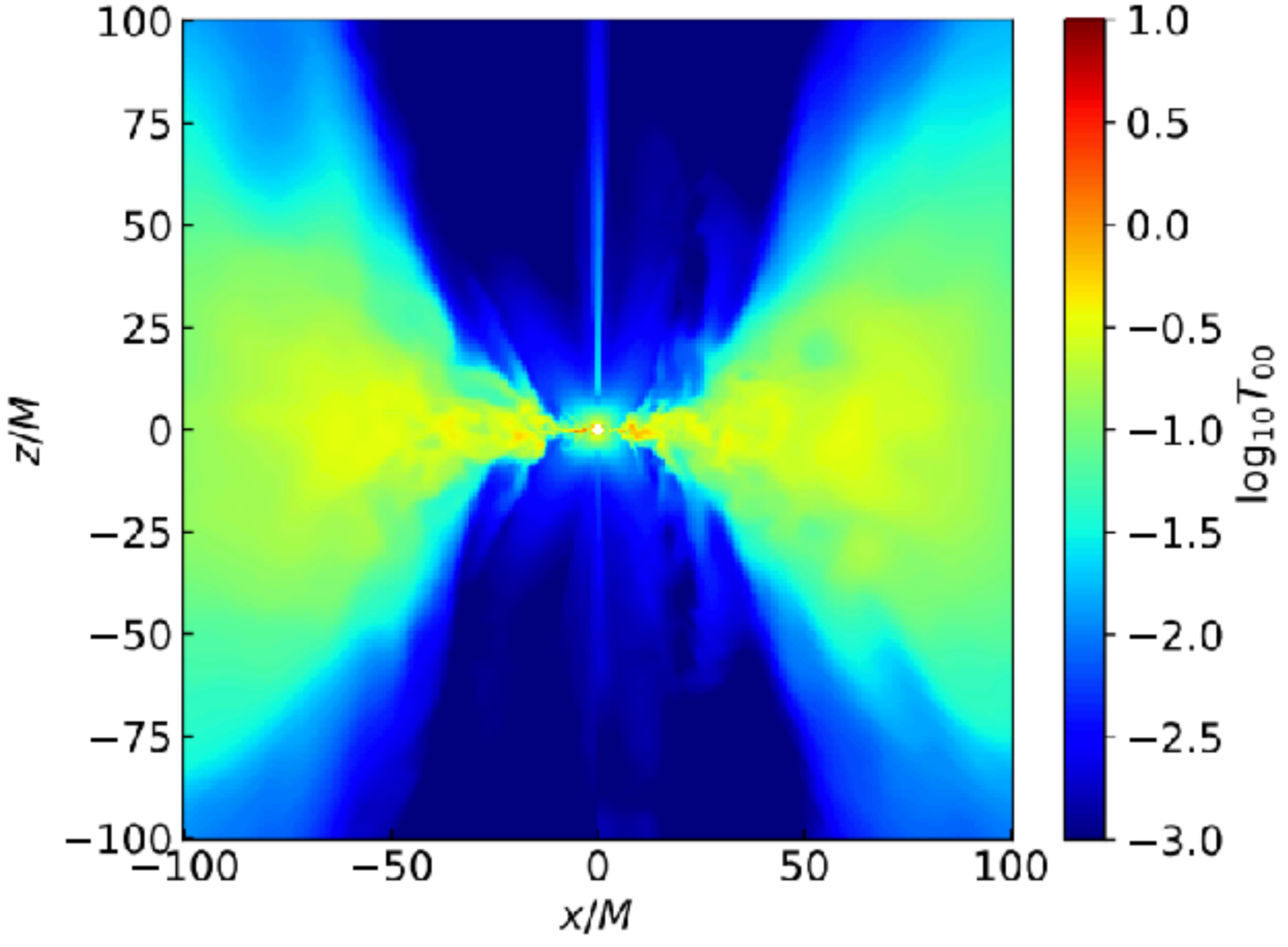
However, if we can trap sufficiently GWs, they exhibit turbulent dynamics

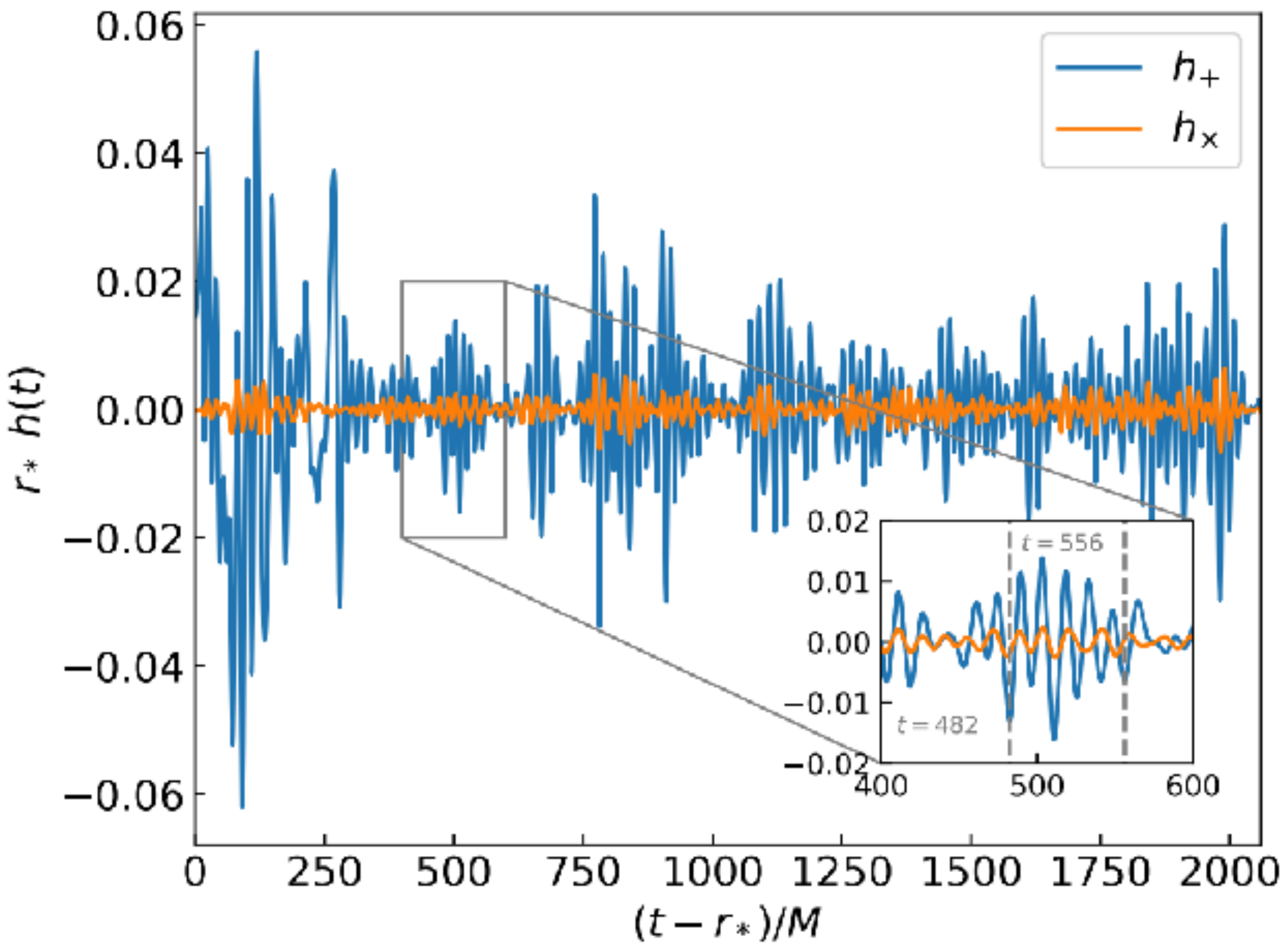
Further nonlinear simulations & analytical work is needed to understand this

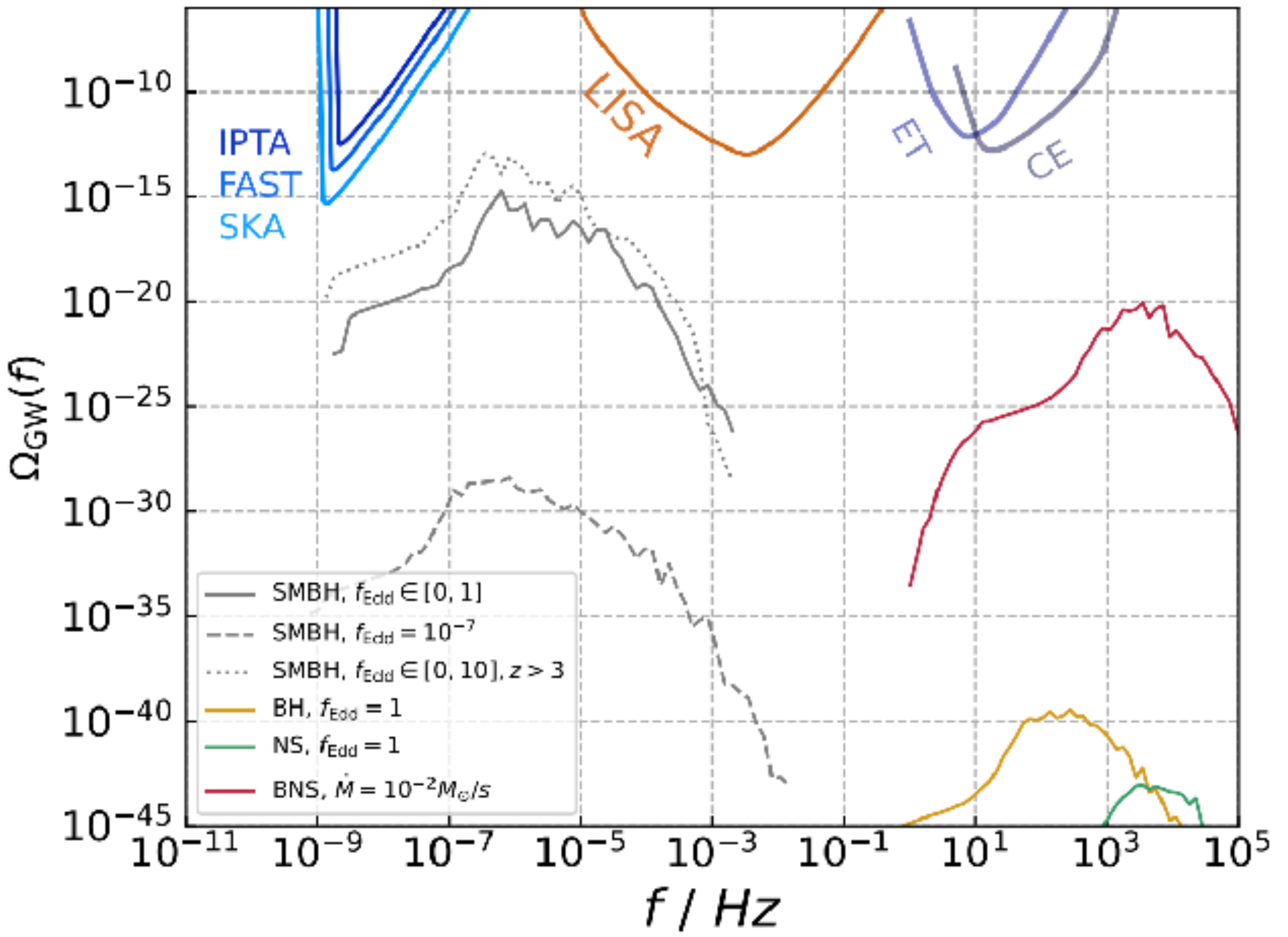
Here comes the Sun

Turbulent systems emit GWs

A turbulent accretion disk emits a turbulent spectrum of GWs





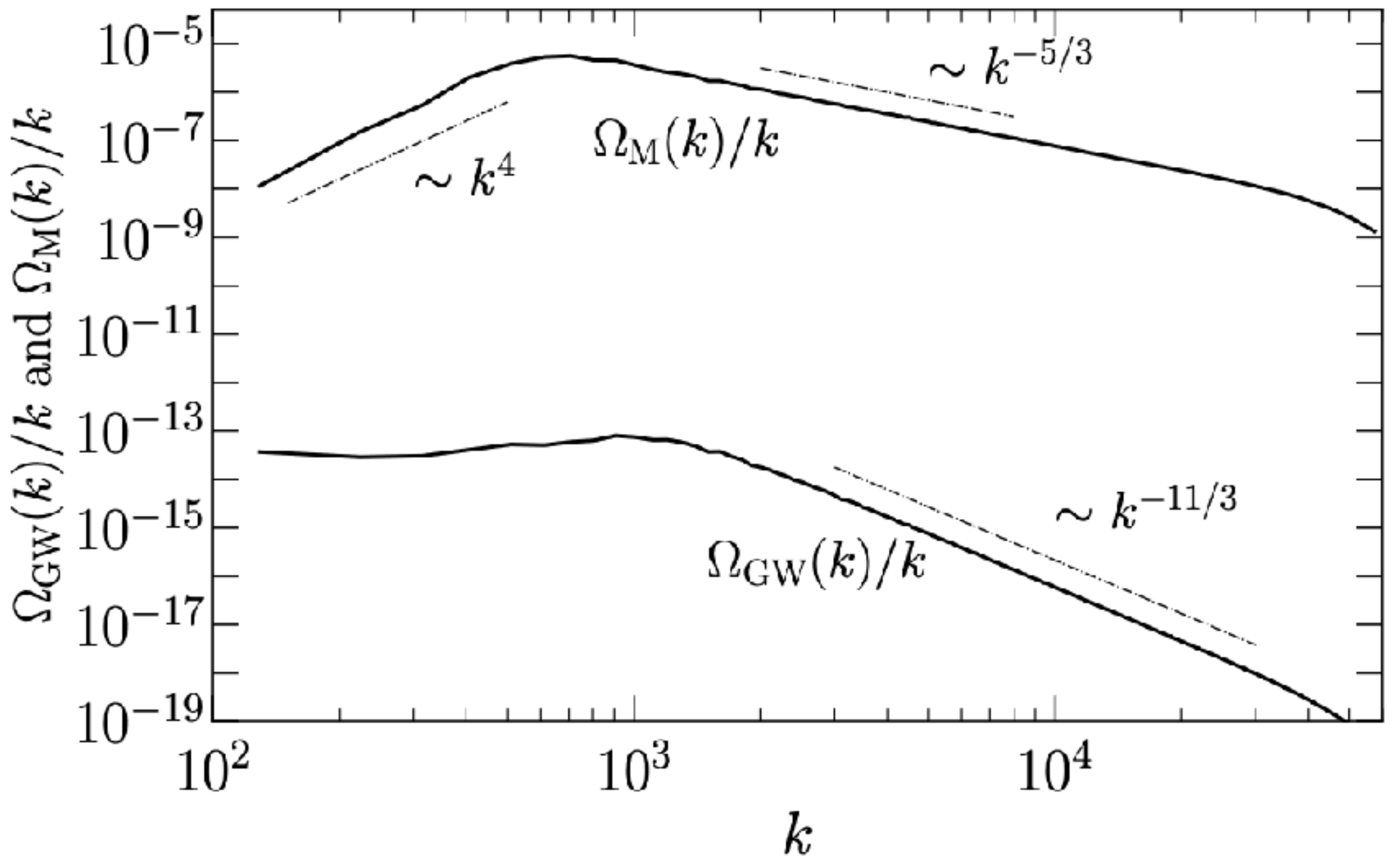


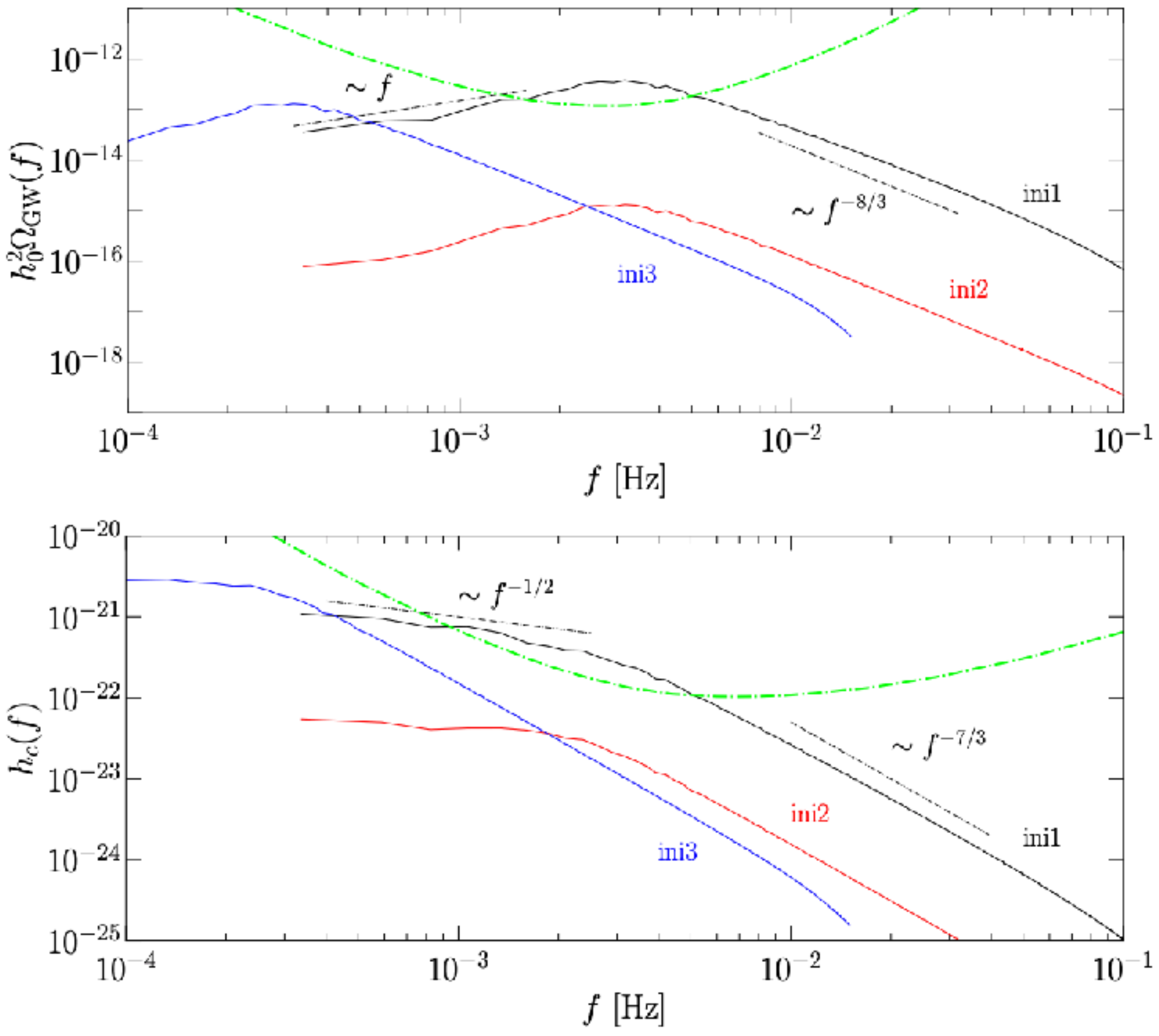
+ BH fingerprints

Chen+ '25

Turbulent systems emit GWs

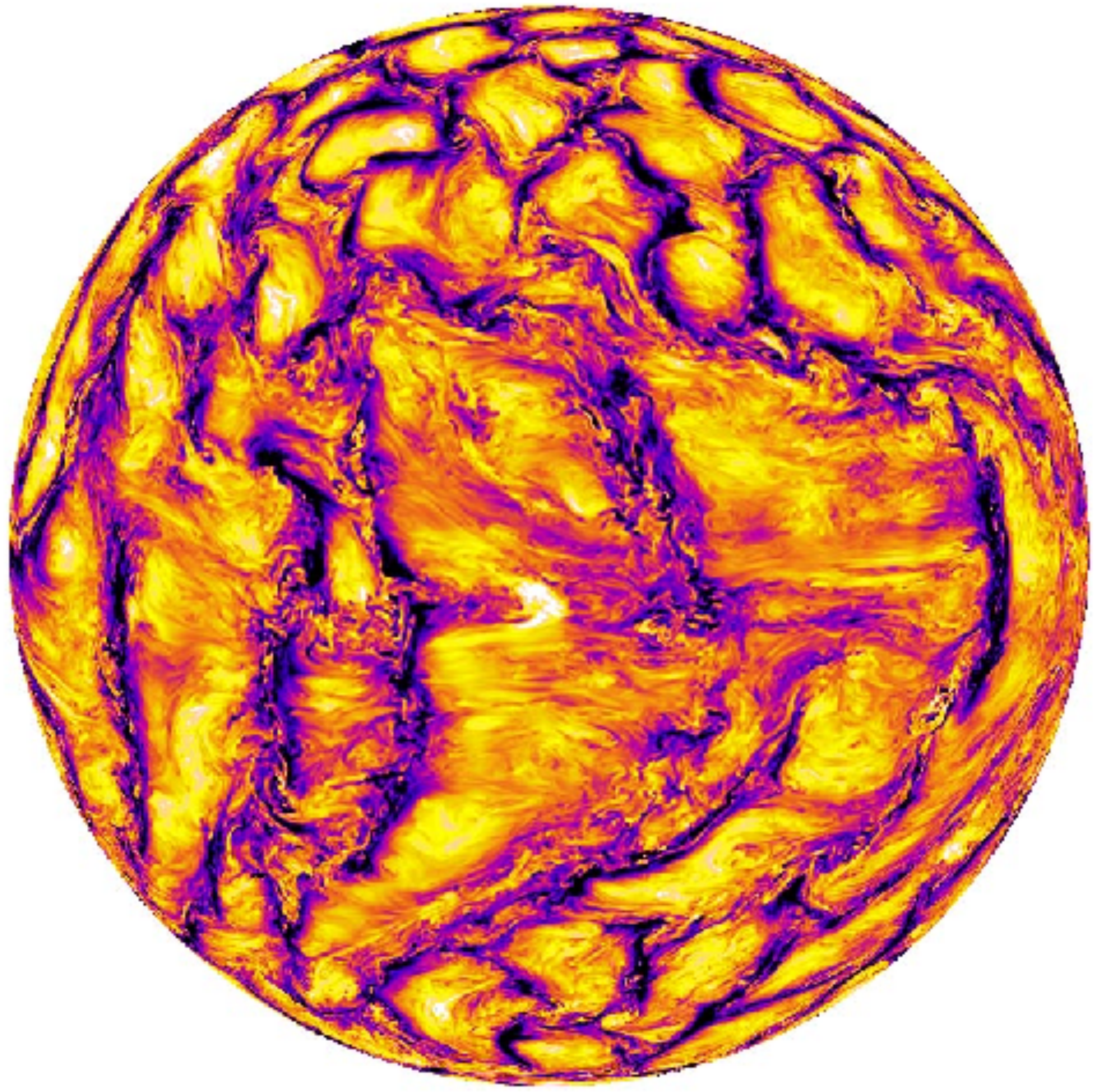
Early-Universe turbulent MHD also emits relic GWs





Roper Pol+ '21

Gravitational wave helioseismology?



$$h^{\mathrm{EM}} = 10^{-22}\left(\frac{d}{AU}\right)\left(\frac{B}{T}\right)^2\left(\frac{R_{\mathrm{Eddy}}}{R_\odot}\right)^5\left(\frac{s}{\tau_{\mathrm{Eddy}}}\right)^2,$$

$$h^{\mathrm{matter}} = 10^{-17}\left(\frac{d}{AU}\right)\left(\frac{\rho}{g\,cm^{-3}}\right)\left(\frac{R_{\mathrm{Eddy}}}{R_\odot}\right)^7\left(\frac{s}{\tau_{\mathrm{Eddy}}}\right)^4.$$

Take-aways

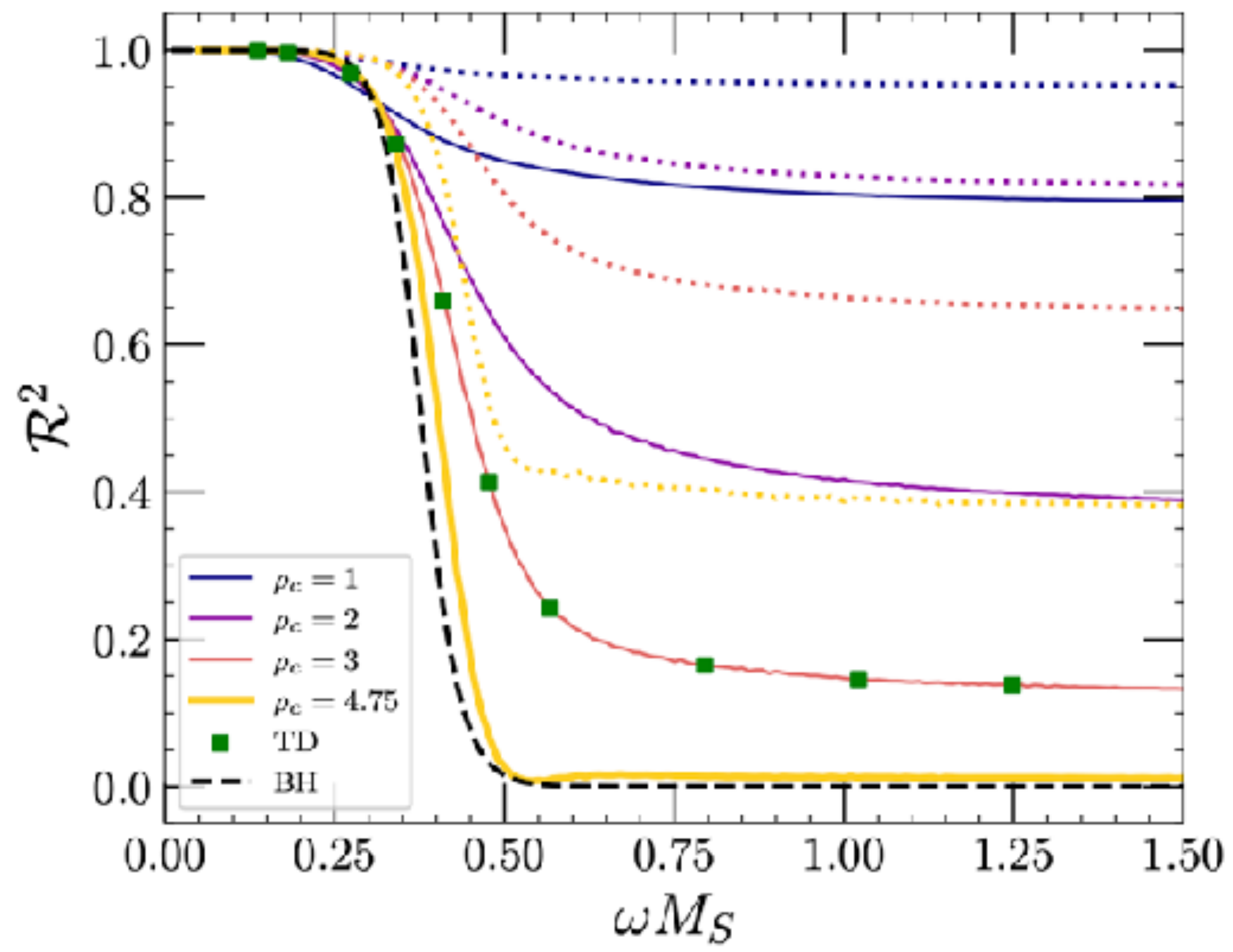
The nonlinear regime of GR and hydro (MHD) share many features

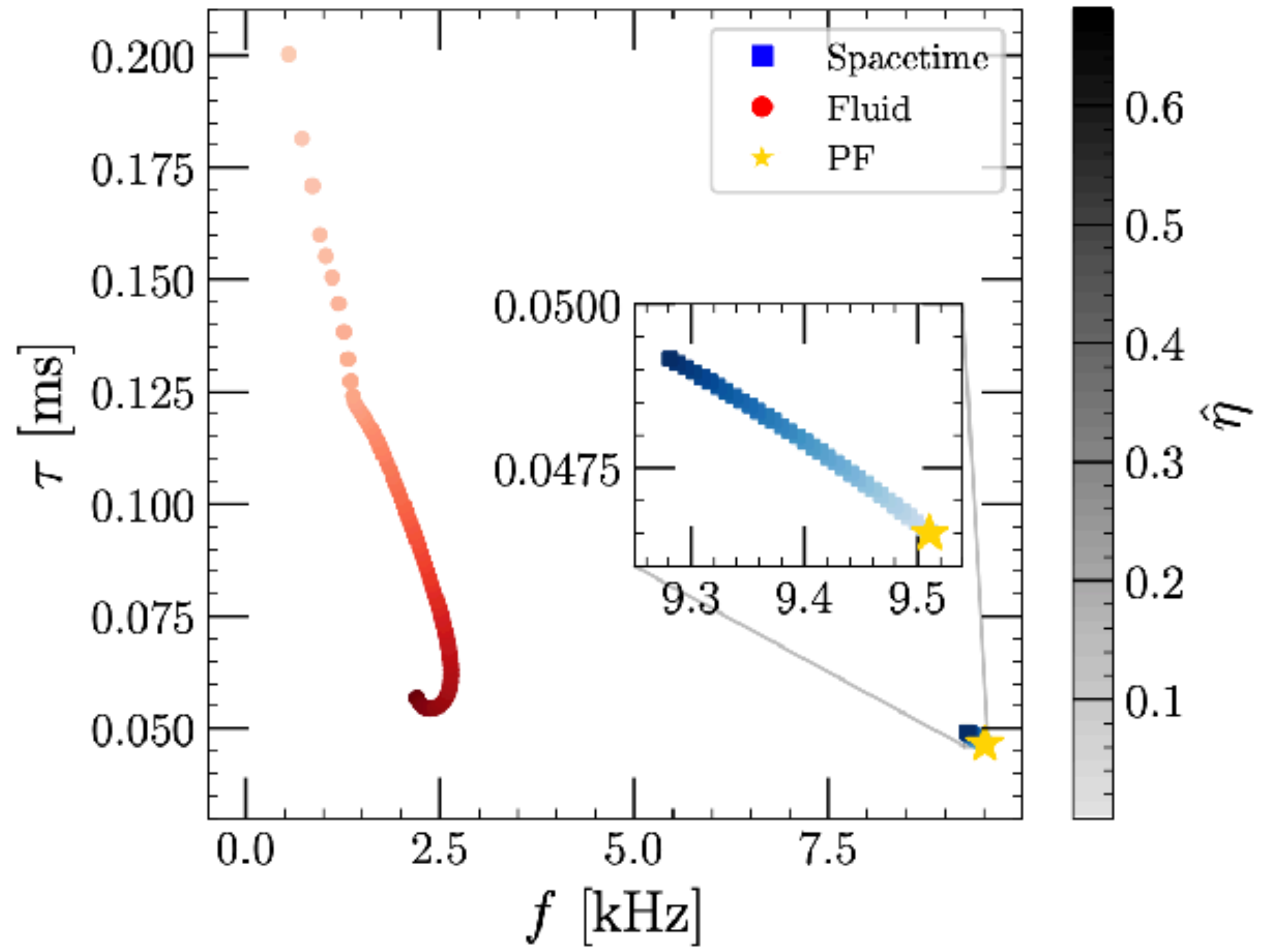
Transfer of numerical expertise has proved very valuable!

We should also put in common theoretical tools!

and keep an open mind

Linear response of BDNK stars





BDNK hydro can be used to model dissipative effects in NS

On-going work in numerical problem. How about the perturbative regime?

Viscosity —> absorption of GWs (mimic BH response)

New dynamical d.o.f. —> new modes with new frees.

Missing!
— Dissipative tidal Love numbers
— Rotation
— Even parity modes
— Realistic EoS

See 2411.16861, 2411.16841